VECTORS

A vector is a quantity that has both direction and magnitude.

Consider a directed line segment with initial point P
And terminal point Q.

Then length of the directed line segment is denoted by $\|\mathrm{PQ}\|$.

If two vectors have the same magnitude and direction, they are equivalent.

The set of all equivalent line segments $\overrightarrow{\mathrm{PQ}}$ is the vector v in the plane.

$$v = \overrightarrow{\mathrm{PQ}}$$

To find the length of a line segment use the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$$

Component Form

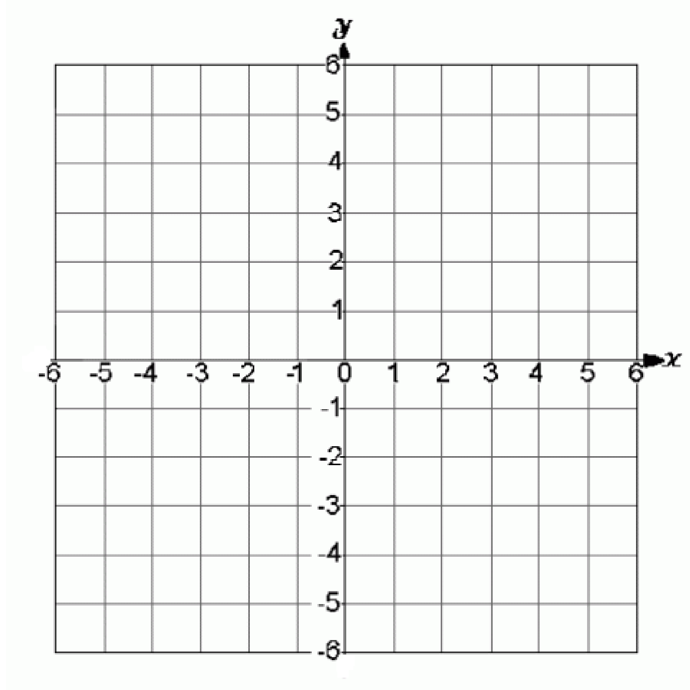
A vector is in standard position if its initial point is on the origin.

A vector whose initial point is on the origin can be identified by the coordinated of its terminal point (v_1, v_2).

The Component Form of a Vector

$$v = \langle v_1, v_2 \rangle$$

Draw $\langle -3, -2 \rangle$



Converting Directed Line Segments to Vector (Component) Form.

Given initial point (p_1, p_2) and terminal point (q_1, q_2)

$$\overrightarrow{pq} = \langle q_1 - p_1, q_2 - p_2 \rangle = \langle v_1, v_2 \rangle = v$$

Magnitude

$$\|v\| = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2} = \sqrt{{v_1}^2 + {v_2}^2}$$

If

Then v is a unit vector.

The

If and only if v is a zero vector where the initial point and terminal points are the same.

Two vectors $u = \langle u_1, u_2 \rangle$ and $v = \langle v_1, v_2 \rangle$ are equal if and only if $u_1 = v_1$ and $u_2 = v_2$

Sample problem

Find the component form and magnitude of vector v given it has an initial point of (4,-7) and a terminal point of (-1,5).

Component Form

$$v = \langle q_1 - p_1 \ , \ q_2 - p_2 \rangle$$

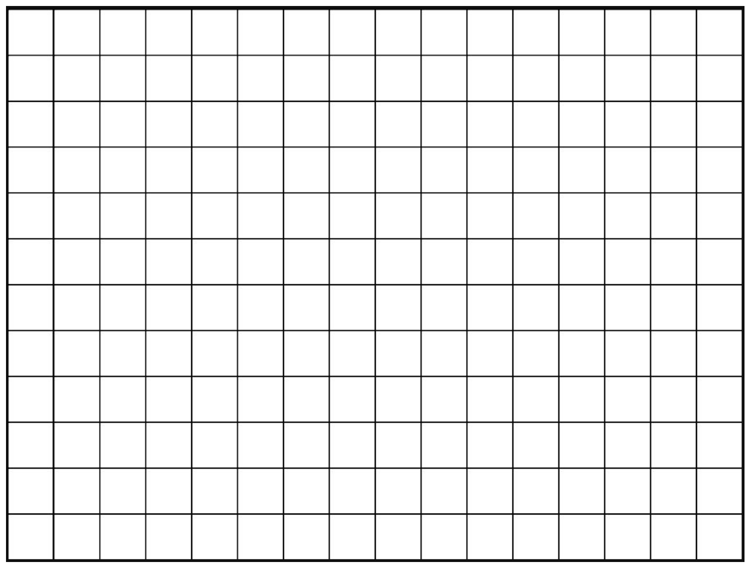
Magnitude

$$\|v\| = \sqrt{v_1^{\ 2} + v_2^{\ 2}}$$

Vector Operations

Scalar multiplication and vector addition.

* The product of vector v and a scalar k is the vector that is $|k|$ times as long as v.



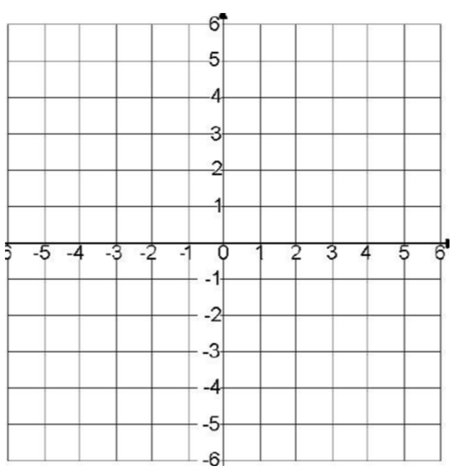
* If k is $(+)$, then kv has the same direction as v.

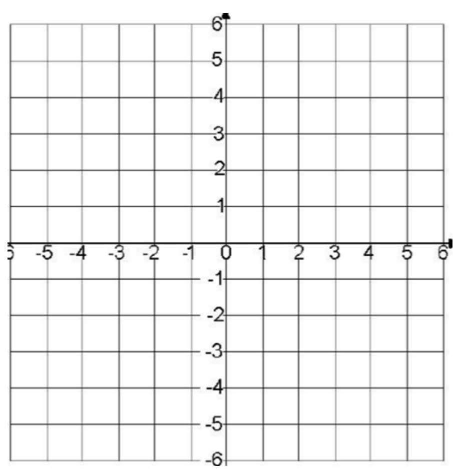
* If k is $(-)$, then kv has the opposite direction as v.

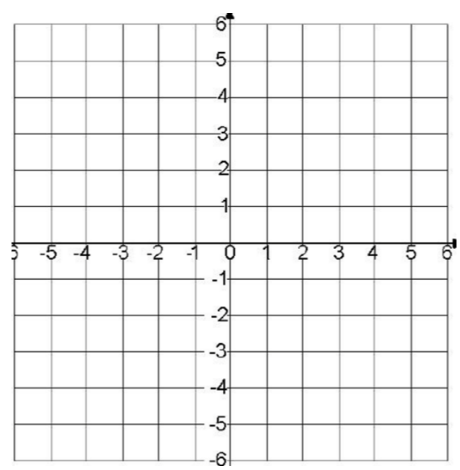
$2v = 2\langle 2,3\rangle$

$v = \langle 2,3\rangle$

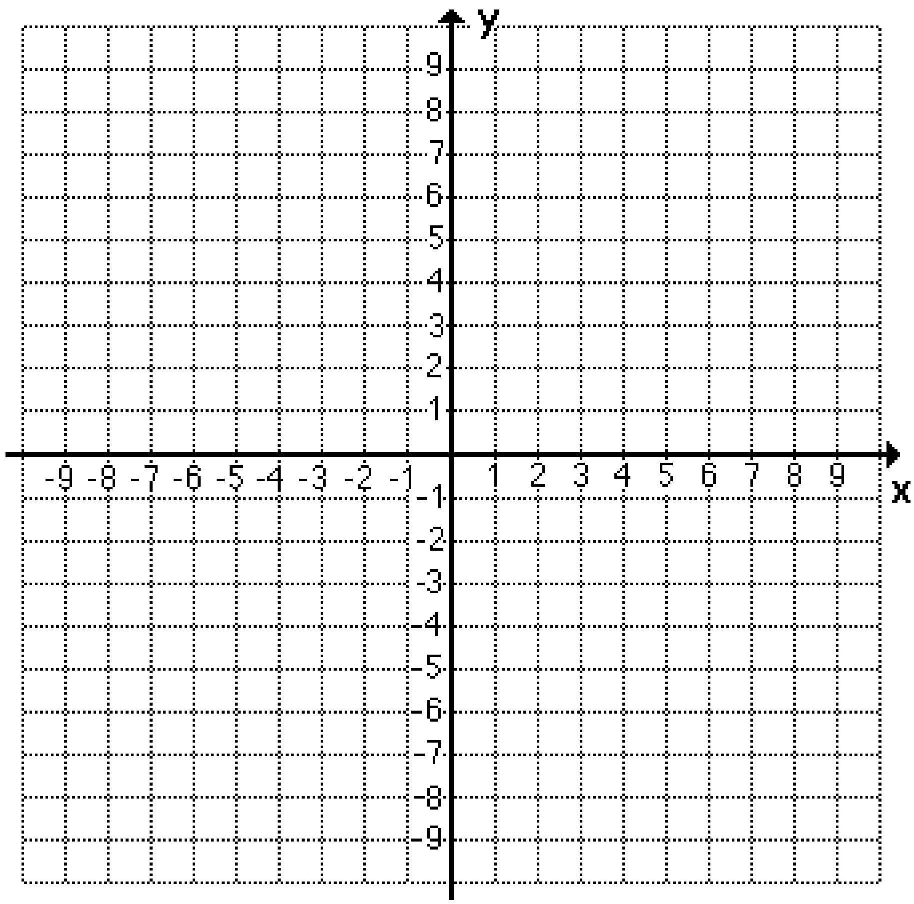
$-v = -1\langle 2,3\rangle$







To add two vectors graphically, reposition them without changing direction or length.



Algebraically

Let $u = \langle u_1, u_2 \rangle$ and $v = \langle v_1, v_2 \rangle$

Then $u + v = \langle u_1 + v_1, u_2 + v_2 \rangle$

Let $v = \langle -2, 5 \rangle$ and $w = \langle 3, 4 \rangle$

Find $v + w$

Find $2v - w$

Vectors Continued

Unit Vectors

$$u = \text{ unit vector } = \frac{u}{\|u\|} = \frac{1}{\|u\|} \cdot u$$

Example:

Find a unit vector in the direction of $v = \langle -2, 5 \rangle$ and verify it has a length of one unit.

Standard Unit Vectors

The standard unit vectors are denoted by:

$$i = \langle 1, 0 \rangle \text{ and } j = \langle 0, 1 \rangle$$

These two vectors can be used to represent <u>any vector</u>.

Example:

Write $\langle -5, 9 \rangle$ as a combination of standard unit vectors.

$$v = \langle v_1, v_2 \rangle = v_1 \langle 1, 0 \rangle + v_2 \langle 0, 1 \rangle$$
$$= v_1 i + v_2 j$$

The scalars v_1 and v_2 are the horizontal and vertical compliments of *v*.

Example:

Let u be a vector with initial point $(-1,3)$ and terminal point $(2,-5)$.
Write u as a linear combination of the standard unit vectors i and j.

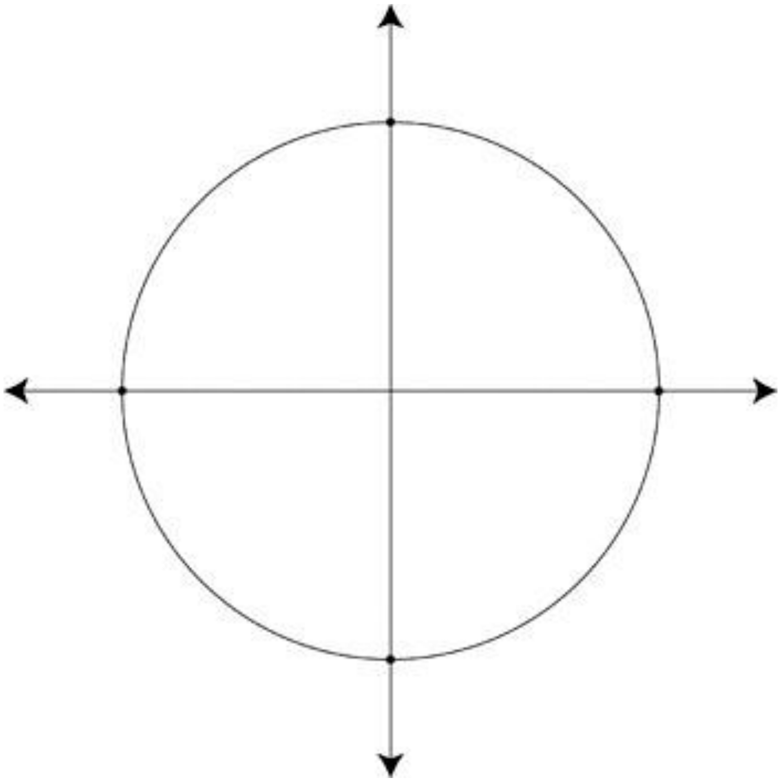
Example:

Given vector $u = -3i + 8j$ and vector $v = 2i - j$,

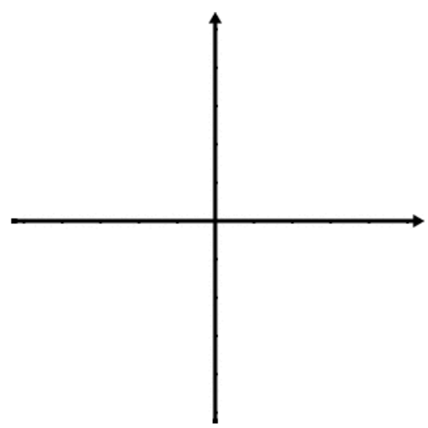
Find $2u - 3v$.

VECTORS CONTINUED

Recall



Direction Angles



If u is a unit vector such that θ is measured counterclockwise:

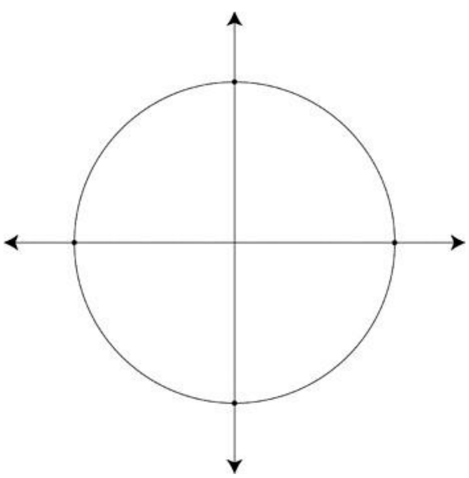
$$u = \langle \cos\theta, \sin\theta \rangle = \cos\theta i + \sin\theta j$$

$$u = \langle \cos\theta, \sin\theta \rangle = \cos\theta i + \sin\theta j$$

* θ is the direction of vector u given u is a unit vector with direction angle θ.

* If v is any vector that makes an angle θ with the x-axis and has the same direction as u ...

$$v = \|v\| \langle \cos\theta, \sin\theta \rangle = \|v\| \cos\theta i + \|v\| \sin\theta j$$



Example:

Vector v has a magnitude of 3 and makes and angle of 30°.

$v = \langle a, b \rangle$

Since $\quad v = ai + bj = \|v\| \cos\theta i + \|v\| \sin\theta j,$

the direction angle θ for v is determined by:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\|v\|\sin\theta}{\|v\|\cos\theta} = \frac{b}{a}$$

So when $v = ai + bj$, then $\tan\theta = \dfrac{b}{a}$.

To find the direction angle θ use $\tan^{-1}\left(\dfrac{b}{a}\right)$.

Example:

Find the direction of the following vectors.

$u = 3i + 3j$ $\qquad\qquad$ $v = 3i - 4j$

*Hint, a diagram will always be helpful to determine the quadrant in which the terminal side of the angle resides.

VECTORS CONTINUED

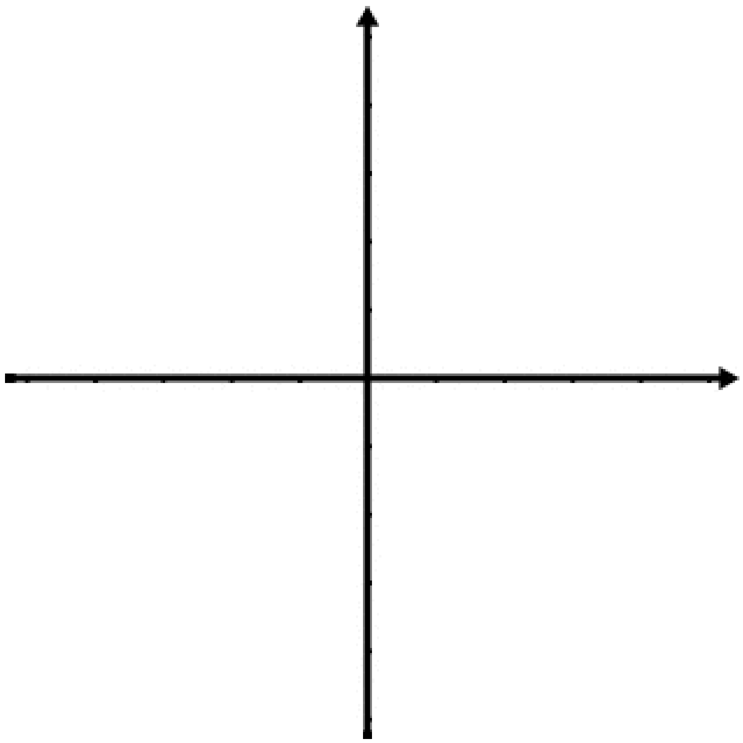
Forces acting on an object

Many of the word problems with vectors involve forces acting on an object.

Example:

Two forces both of 200lbs act on an object. The angle between the forces is 80°.

Find the direction and magnitude of the resultant force.



$\|v\|\cos\theta i + \|v\|\sin\theta j = ai + bj$

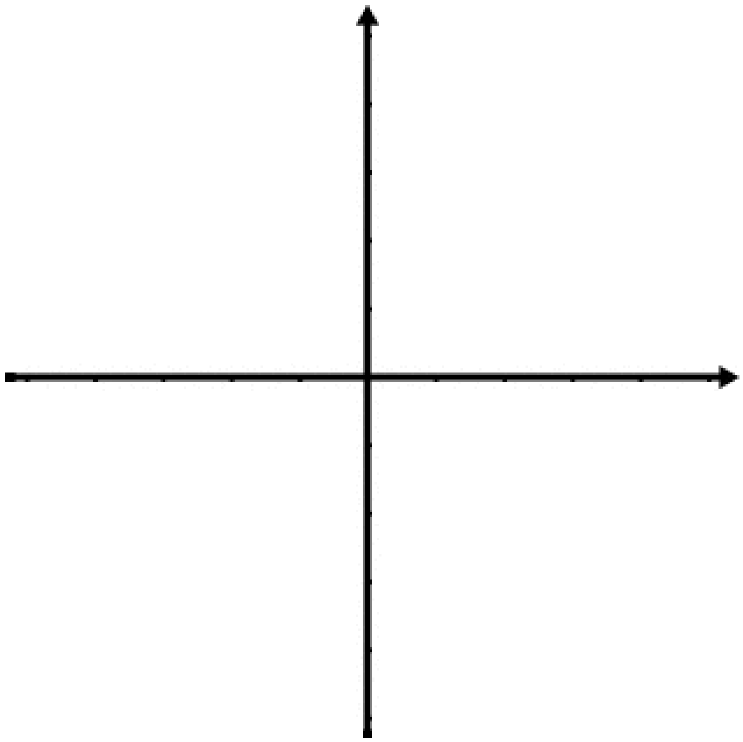
when $v = ai + bj$, then $\tan\theta = \dfrac{b}{a}$.

To find the direction angle θ use $\tan^{-1}\left(\dfrac{b}{a}\right)$.

Example:

Two forces both of 200lbs act on an object. The angle between the forces is 40°.

Find the direction and magnitude of the resultant force.



$\|v\|\cos\theta i + \|v\|\sin\theta j = ai + bj$

when $v = ai + bj$, then $\tan\theta = \frac{b}{a}$.

To find the direction angle θ use $\tan^{-1}\left(\frac{b}{a}\right)$.

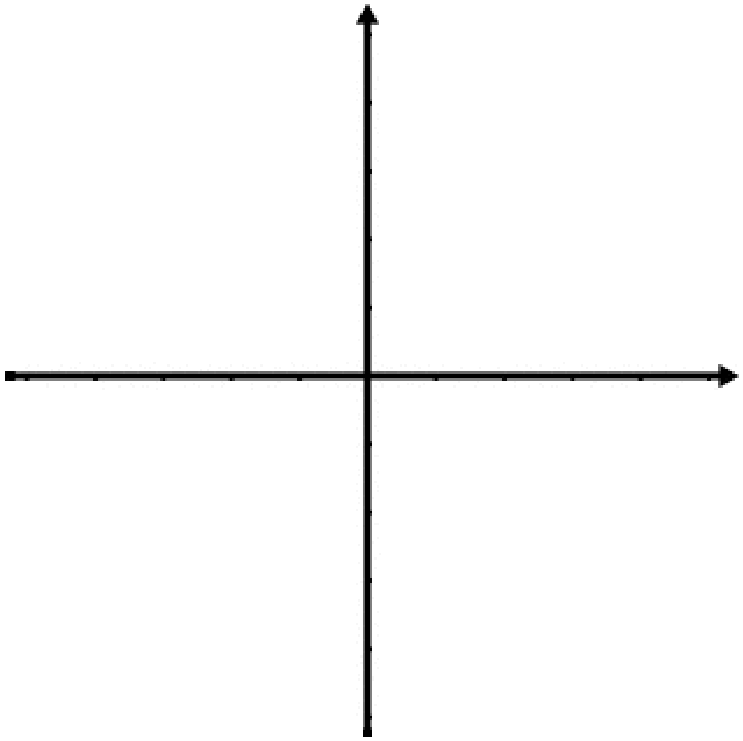
***As the angle between the two forces decreases, the magnitude of the resultant force increases!**

Example:

Two forces one of 150 lbs and the other of 200lbs act on an object.

The angle between the forces is 80°.

Find the direction and magnitude of the resultant force.



$$\|v\|\cos\theta i+\|v\|\sin\theta j=ai+bj$$

when $v=ai+bj$, then $\tan\theta=\frac{b}{a}$.

To find the direction angle θ use $\tan^{-1}\left(\frac{b}{a}\right)$.

***As the difference in forces acting on an object becomes more pronounced, the direction of the resultant force will move towards the greater of the two forces.**