Ch 5 Review Worksheet

Verify (Prove one side)

1. $\dfrac{1}{1-\sin^2 x} = 1+\tan^2 x$

$" = 1 + \dfrac{\sin^2 x}{\cos^2 x}$

$" = \dfrac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$\dfrac{1}{\cos^2 x} = \dfrac{1}{\cos^2 x}$

2. $\dfrac{\cos^2 x}{1-\sin x} = \left(\dfrac{\cos x}{\sec x - \tan x}\right)\left(\dfrac{\sec x + \tan x}{\sec x + \tan x}\right)$

$\dfrac{1-\sin^2 x}{1-\sin x} = \dfrac{\cos x\,(\sec x + \tan x)}{\sec^2 x - \tan^2 x}$

$\dfrac{(1+\sin x)(1-\sin x)}{1-\sin x} = \cos x\left(\dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x}\right)$

$" = \cos x\left(\dfrac{1+\sin x}{\cos x}\right)$

$1 + \sin x = 1 + \sin x$

3. $\dfrac{\sin 2x}{1+\cos 2x} = \tan x$

$\dfrac{2\sin x\cos x}{1+\cos^2 x - \sin^2 x} =$

$\dfrac{2\sin x\cos x}{(1-\sin^2 x)+\cos^2 x} =$

$\dfrac{2\sin x\cos x}{\cos^2 x + \cos^2 x} =$

$\dfrac{\cancel{2}\sin x\,\cancel{\cos x}}{\cancel{2}\cos^{\cancel{2}} x} =$

$\tan x = \tan x$

Solve $[0, 2\pi)$

4. $\cos x\sin x - \sin x = 0$

$\sin x\,(\cos x - 1) = 0$

$\sin x = 0 \qquad \cos x = 1$

$x = 0, \pi$

5. $2\sin^2 x - 5\sin x + 2 = 0$

$(2\sin x - 1)(\sin x - 2) = 0$

$2\sin x - 1 = 0 \qquad \sin x - 2 = 0$

$2\sin x = 1 \qquad \sin x = 2$

$\sin x = \frac{1}{2} \qquad \cancel{\sin x = 2}$

not possible

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

6. $\tan^3 x + \tan^2 x - 3\tan x - 3 = 0$

$\tan^2 x(\tan x + 1) - 3(\tan x - 1) = 0$

$(\tan^2 x - 3)(\tan x + 1) = 0$

$\tan^2 x - 3 = 0 \qquad \tan x + 1 = 0$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3} \qquad \tan x = -1$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{3\pi}{4}, \frac{7\pi}{4}$

7. $\sin 2x - \cos x = 0$

$2\sin x\cos x - \cos x = 0$

$\cos x\,(2\sin x - 1) = 0$

$2\sin x - 1 = 0$

$\cos x = 0 \qquad \sin x = \frac{1}{2}$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

Find the exact value w/o calculators

$45^\circ - 30^\circ = 15^\circ$

8. $\cos 15^\circ$

$\cos(45^\circ - 30^\circ)$

$\cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$\cos 15^\circ = \frac{\sqrt{6} + \sqrt{2}}{4}$

$2\sin x \cos x = \sin 2x$

9. $2\sin\frac{\pi}{12}\cos\frac{\pi}{12}$

$\sin(2)\left(\frac{\pi}{12}\right)$

$\sin\frac{\pi}{6}$

$\sin\frac{\pi}{6} = \frac{1}{2}$

10. $\frac{\tan 66^\circ - \tan 6^\circ}{1 + \tan 66^\circ \tan 6^\circ}$

$\tan(66^\circ - 6^\circ)$

$\tan 60^\circ$

$\tan 60^\circ = \sqrt{3}$

11. $\cos^2\left(\frac{\pi}{8}\right) - \sin^2\left(\frac{\pi}{8}\right)$

$\cos^2 x - \sin^2 x = \cos 2x$

$\cos(2)\left(\frac{\pi}{8}\right)$

$\cos\frac{\pi}{4}$

$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

12. $\csc 15^\circ$

$\sin 15^\circ = \sin(45^\circ - 30^\circ)$

$\sin 15^\circ = \sin 45 \cos 30 - \cos 45 \sin 30$

$\sin 15^\circ = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$\sin 15^\circ = \frac{\sqrt{6} - \sqrt{2}}{4}$

$\csc 15^\circ = \left(\frac{4}{\sqrt{6} - \sqrt{2}}\right)\left(\frac{\sqrt{6} + \sqrt{2}}{\sqrt{6} + \sqrt{2}}\right)$

$\csc 15^\circ = \frac{4(\sqrt{6} + \sqrt{2})}{6 - 2}$

$\csc 15^\circ = \sqrt{6} + \sqrt{2}$

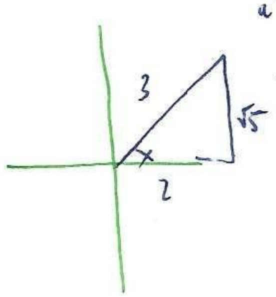
Find the following given that $\sec x = \frac{3}{2}$, $\csc y = 3$, $\angle x$ **and** $\angle y$ **are in Quadrant I.**

$\angle x$

$\sin x = \frac{\sqrt{5}}{3}$

$\cos x = \frac{2}{3}$

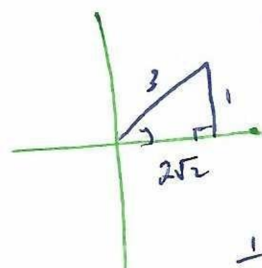
$\tan x = \frac{\sqrt{5}}{2}$



$a^2 + 4 = 9$

$a^2 = 5$

$a = \sqrt{5}$



$\angle y$

$a^2 + 1 = 9$

$a^2 = 8$

$a = 2\sqrt{2}$

$\sin y = \frac{1}{3}$

$\cos y = \frac{2\sqrt{2}}{3}$

$\frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$ $\quad \tan y = \frac{\sqrt{2}}{4}$

13. $\sin(x + y)$

$\sin x \cos y + \cos x \sin y$

$\left(\frac{\sqrt{5}}{3}\right)\left(\frac{2\sqrt{2}}{3}\right) + \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)$

$\frac{2\sqrt{10}}{9} + \frac{2}{9}$

$\sin(x+y) = \frac{2\sqrt{10} + 2}{9}$

14. $\cos(x - y)$

$\cos x \cos y + \sin x \sin y$

$\left(\frac{2}{3}\right)\left(\frac{2\sqrt{2}}{3}\right) + \left(\frac{\sqrt{5}}{3}\right)\left(\frac{1}{3}\right)$

$\frac{4\sqrt{2}}{9} + \frac{\sqrt{5}}{9}$

$\cos(x-y) = \frac{4\sqrt{2} + \sqrt{5}}{9}$

15. $\tan(x + y)$ $= \frac{4\sqrt{5} + 2\sqrt{2}}{8 - \sqrt{10}}$

$\frac{\tan x + \tan y}{1 - \tan x \tan y}$

$\frac{\left(\frac{2}{2}\right)\left(\frac{\sqrt{5}}{2}\right) + \frac{\sqrt{2}}{4}}{1 - \left(\frac{\sqrt{5}}{2}\right)\left(\frac{\sqrt{2}}{4}\right)}$

$\frac{2\sqrt{5} + \sqrt{2}}{4} \div \left(\frac{8}{8} - \frac{\sqrt{10}}{8}\right)$

$\frac{2\sqrt{5} + \sqrt{2}}{\cancel{4}} \cdot \frac{\overset{2}{\cancel{8}}}{8 - \sqrt{10}}$

$\sin x = \frac{\sqrt{5}}{3}$

$\cos x = \frac{2}{3}$

$\tan x = \frac{\sqrt{5}}{2}$

$\sin y = \frac{1}{3}$

$\cos y = \frac{2\sqrt{2}}{3}$

$\tan y = \frac{\sqrt{2}}{4}$

16. $\sin 2x$

$\sin 2x = 2\sin x \cos x$

$= 2\left(\frac{\sqrt{5}}{3}\right)\left(\frac{2}{3}\right)$

$\sin 2x = \frac{4\sqrt{5}}{9}$

17. $\cos\frac{y}{2}$

$\cos\frac{y}{2} = \pm\sqrt{\frac{1+\cos y}{2}}$

$= \sqrt{\frac{1+\frac{2\sqrt{2}}{3}}{2}}$

$\angle y$ is in quad I so $\frac{y}{2}$ is a/so in Quad I

$= \sqrt{\frac{3+2\sqrt{2}}{3} \div 2}$

$= \sqrt{\frac{3+2\sqrt{2}}{3} \cdot \frac{1}{2}}$

$\cos\frac{y}{2} = \sqrt{\frac{3+2\sqrt{2}}{6}}$

18. $\tan\frac{y}{2}$

$\sin y = \frac{1}{3} \quad \cos y = \frac{2\sqrt{2}}{3}$

$\tan\frac{y}{2} = \frac{1-\cos y}{\sin y}$

$= \frac{1-\frac{2\sqrt{2}}{3}}{\frac{1}{3}}$

$= \frac{3-2\sqrt{2}}{3} \div \frac{1}{3}$

$= \frac{3-2\sqrt{2}}{3} \cdot \frac{3}{1}$

$\tan\frac{y}{2} = 3-2\sqrt{2}$

19. $\sin(x-y)$

$\sin x \cos y - \cos x \sin y$

$\left(\frac{\sqrt{5}}{3}\right)\left(\frac{2\sqrt{2}}{3}\right) - \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)$

$\frac{2\sqrt{10}}{9} - \frac{2}{9}$

$\sin(x-y) = \frac{2\sqrt{10}-2}{9}$

Solve

20. $\tan^2 2x - 9 = 0$

$\sqrt{\tan^2 2x} = \sqrt{9}$

$\tan 2x = \pm 3$

$\tan^{-1}(-3) \qquad \tan^{-1}(3)$

$\frac{2x}{2} = \frac{1.8926 + n\pi}{2} \qquad \frac{2x}{2} = \frac{1.2490 + n\pi}{2}$

$x = 0.9463 + \frac{n\pi}{2} \qquad 0.6245 + \frac{n\pi}{2}$

$x = 0.9463 + \frac{n\pi}{2},\ 0.6245 + \frac{n\pi}{2}$

21. $2\cos 3x - 1 = 0$

$2\cos 3x = 1$

$\cos 3x = \frac{1}{2}$

$\frac{1}{3}(3x) = \frac{1}{3}\left(\frac{\pi}{3} + 2n\pi\right)$

$\frac{1}{3}(3x) = \frac{1}{3}\left(\frac{5\pi}{3} + 2n\pi\right)$

$x = \frac{\pi}{9} + \frac{2n\pi}{3},\ \frac{5\pi}{9} + \frac{2n\pi}{3}$

$x = \frac{\pi}{9} + \frac{2n\pi}{3},\ \frac{5\pi}{9} + \frac{2n\pi}{3}$

or

$\frac{\pi}{9} + \frac{6n\pi}{9},\ \frac{5\pi}{9} + \frac{6n\pi}{9}$

22. $\sqrt{2}\sin 2x + 1 = 0$

$\sqrt{2}\sin 2x = -1$

$\sin 2x = -\frac{1}{\sqrt{2}}$

$\sin 2x = -\frac{\sqrt{2}}{2}$

$\frac{2x}{2} = \frac{1}{2}\left(\frac{5\pi}{4} + 2n\pi\right) \qquad \frac{2x}{2} = \frac{1}{2}\left(\frac{7\pi}{4} + 2n\pi\right)$

$x = \frac{5\pi}{8} + \frac{8n\pi}{8} \qquad \frac{7\pi}{8} + \frac{8n\pi}{8}$

$x = \frac{5\pi}{8} + n\pi,\ \frac{7\pi}{8} + n\pi$

or

$\frac{5\pi}{8} + \frac{8n\pi}{8},\ \frac{7\pi}{8} + \frac{8n\pi}{8}$