Left and Right Behaviors of Polynomial Functions

As we graphed various functions, you should have noticed something about the graph of a polynomial function of an even degree versus the graph of a polynomial function of an odd degree. Think of a parabola versus a cubic function. The left and right behaviors of polynomial functions are pretty simple to memorize.

If the degree of the polynomial is even, the graph of the function will have either "both sides up", or "both sides down."

If the degree of the polynomial is odd, the graph of the function will have one side up and one side down.

As to which side is up and which is down, that all depends on the leading coefficient.

Refer to the following.

	Even Degree	*Odd Degree*
+ leading coefficient	↑↑	↓↑
- leading coefficient	↓↓	↑↓

Therefore, a 7th degree polynomial function having a leading coefficient that is negative, will rise on the left, and fall to the right.

In contrast, if the 7th degree polynomial has a positive leading coefficient, the graph of the function will fall on the left, and rise on the right hand side.

These rules are for polynomial functions in a single variable only!

When we begin to graph these polynomial functions, the first step will be to find all zeros of the function. The x intercepts have been found, plot them on the x axis, and refer to the two intercepts on the ends. At this point, use the rules for left and right behaviors of functions to draw a portion of the graph.

<u>Graphing Polynomial Functions</u>

When graphing a polynomial function, there are a series of steps to follow.

1. ***Find all zeros of the function. This will give the x intercepts of the function***
2. ***Plot all x intercepts for the function on the x axis.***
3. ***Using the properties of polynomial functions, determine the left and right behaviors of the function, and draw those segments.***
4. ***Substitute zero for x, and find the y intercept of the function.***
5. ***Using the graphing calculator, find the maxima and minima between each x intercept.***
6. ***Draw the rest of the function, making sure the maxima and minima are in their appropriate locations.***

The first step is the most time consuming, as the rational zero test and other methods are used to find all real zeros of the function. Depending on the degree of the function, there may be quite a few x intercepts to find. Imaginary solutions to the equation are not x intercepts. They will not be on the graph of the function.

The two most important properties of polynomial used to graph, are the left and right behaviors of a polynomial function, and the rule regarding the number of turns of a polynomial function.

	Even Degree	***Odd Degree***
+ leading coefficient	↑↑	↓↑
- leading coefficient	↓↓	↑↓

Any polynomial function to the nth degree, has <u>at most</u> n-1 turns.

This means a 5^{th} degree polynomial function will have at most 4 turns. It does not have to have that many, but it can have no more than 4 turns.

Sketch the graph of each of the following polynomial functions. **(Label all x intercepts, y intercepts, maxima, minima, and identify the range and domain.) Scale the graphs as needed.**

1) $f(x) = x^4 - 10x^2 + 24$

2) $f(x) = \frac{1}{2}(x^2 - 2x + 15)$

3) $f(x) = -(x-1)^2(x+4)$

4) $f(x) = \frac{1}{2}x(x-4)(x+7)$

5) $f(x) = -(x+3)^3(x-1)(x-5)$

6) $f(x) = -3x(x-2)^2(x+5)$

7) $f(x) = \frac{1}{4}(x+5)^3(x+1)(x-3)$

8) $f(x) = -x^2(x+3)(x-2)(x-6)^2$

9) $f(x) = -(x+3)^3(x-5)^2$

10) $f(x) = x(x-3)^2(x+3)^2$