

Trigonometric Functions Study Guide

SINE FUNCTIONS

Standard Form: $y = a \sin(bx - c) + d$

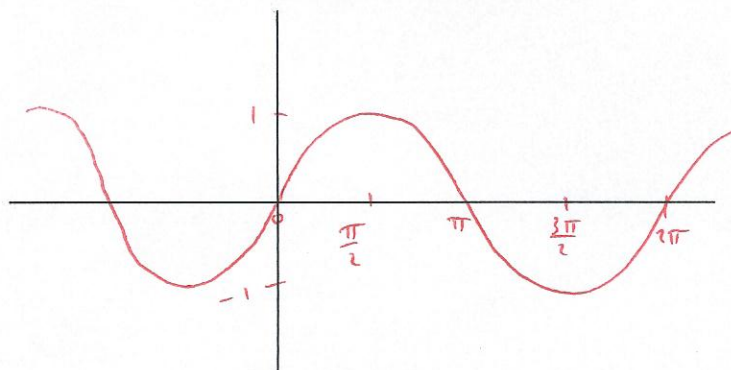
Amplitude: $|a|$

Period: $\frac{2\pi}{|b|}$

Phase Shift: $-\frac{c}{b}$

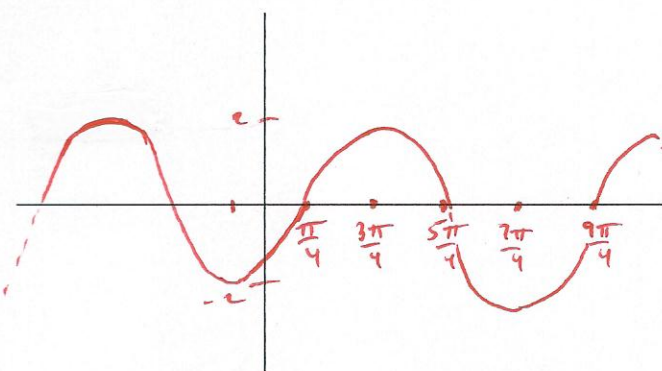
Initial Interval: $0 \leq bx - c \leq 2\pi$

Parent Function $y = \sin x$



Practice Problems

$$y = 2 \sin\left(x - \frac{\pi}{4}\right)$$



Amplitude: 2

Period: 2π

Phase Shift: $\frac{\pi}{4}$

Initial Interval: $\frac{\pi}{4} \leq x \leq \frac{9\pi}{4}$

$$\begin{array}{c} \text{I I} \\ 0 \leq x - \frac{\pi}{4} \leq 2\pi \\ +\frac{\pi}{4} \quad +\frac{\pi}{4} \quad +\frac{\pi}{4} \\ \frac{\pi}{4} \leq x \leq \frac{9\pi}{4} \end{array}$$

$$\frac{\text{Period}}{4} = \frac{2\pi}{4} = \boxed{\frac{\pi}{2}} \text{ add}$$

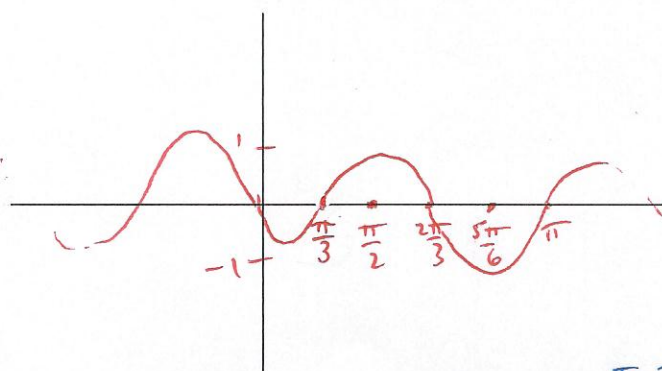
$$\frac{\pi}{4} + \frac{2\pi}{4} = \frac{3\pi}{4}$$

$$\frac{3\pi}{4} + \frac{2\pi}{4} = \frac{5\pi}{4}$$

$$\frac{5\pi}{4} + \frac{2\pi}{4} = \frac{7\pi}{4}$$

$$\frac{7\pi}{4} + \frac{2\pi}{4} = \frac{9\pi}{4}$$

$$y = \sin(3x - \pi)$$



Amplitude: 1

Period: $\frac{2\pi}{3}$

Phase Shift: $\frac{\pi}{3}$

Initial Interval: $\frac{\pi}{3} \leq x \leq \pi$

$$\begin{array}{c} \text{I I} \\ 0 \leq 3x - \pi \leq 2\pi \\ +\pi \quad +\pi \quad +\pi \\ \frac{\pi}{3} \leq \frac{3x}{3} \leq \frac{3\pi}{3} \\ \frac{\pi}{3} \leq x \leq \pi \end{array}$$

$$\frac{\text{Period}}{4} = \frac{2\pi}{3} \div 4 = \frac{2\pi}{3} \cdot \frac{1}{4} = \frac{1}{2} \cdot \frac{\pi}{3} = \boxed{\frac{\pi}{6}} \text{ Add}$$

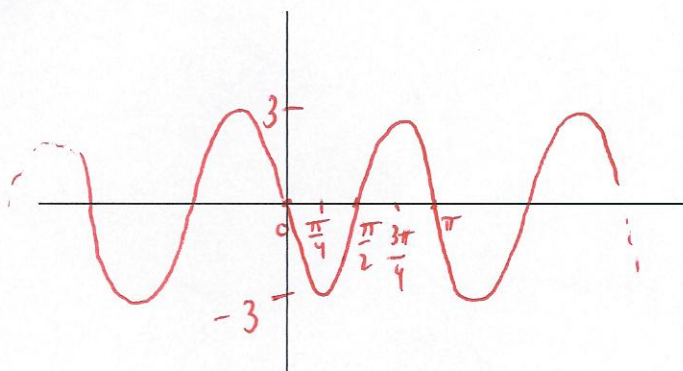
$$\frac{2\pi}{6} + \frac{\pi}{6} = \frac{\pi}{2}$$

$$\frac{3\pi}{6} + \frac{\pi}{6} = \frac{2\pi}{3}$$

$$\frac{4\pi}{6} + \frac{\pi}{6} = \frac{5\pi}{6}$$

$$\frac{5\pi}{6} + \frac{\pi}{6} = \pi$$

$$y = -3 \sin(2x)$$



Amplitude: 3

Period: $\frac{2\pi}{2} = \pi$

Phase Shift: none

Initial Interval: $0 \leq x \leq \pi$

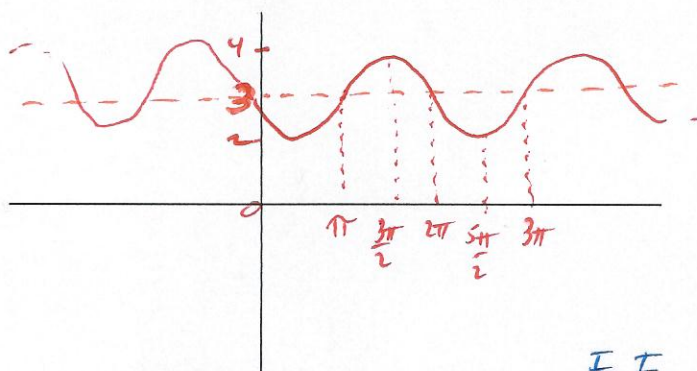
period $\frac{\pi}{4}$ $\boxed{\frac{\pi}{4}}$ add

$$0 + \frac{\pi}{4} = \frac{\pi}{4}$$

$$\frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$$

$$\frac{2\pi}{4} + \frac{\pi}{4} = \frac{3\pi}{4}$$

$$y = \sin(x - \pi) + 3$$



Amplitude: 1

Period: 2π

Phase Shift: π

Initial Interval: $\pi \leq x \leq 3\pi$

period $\frac{2\pi}{4} = \boxed{\frac{\pi}{2}}$ add

$$\pi + \frac{\pi}{2} = \frac{3\pi}{2}$$

$$\frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$$

$$\frac{4\pi}{2} + \frac{\pi}{2} = \frac{5\pi}{2}$$

$$\frac{5\pi}{2} + \frac{\pi}{2} = 3\pi$$

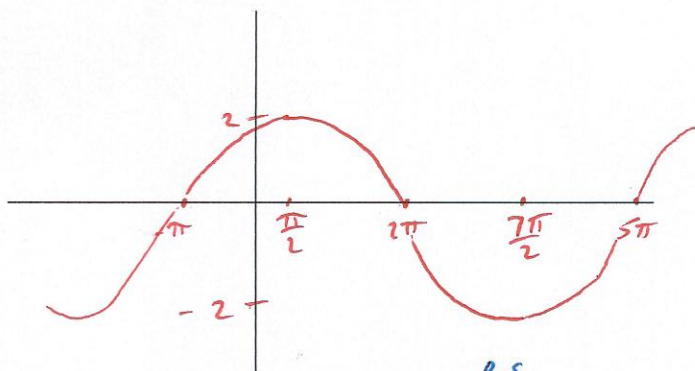
I, I

$$0 \leq x - \pi \leq 2\pi$$

$$+\pi \quad +\pi \quad +\pi$$

$$\pi \leq x \leq 3\pi$$

$$y = 2 \sin\left(\frac{x}{3} + \frac{\pi}{3}\right)$$



Amplitude: 2

Period: $2\pi \div \frac{1}{3} = 6\pi$

Phase Shift: $-\pi$

Initial Interval: $-\pi \leq x \leq 5\pi$

period $\frac{6\pi}{4} = \boxed{\frac{3\pi}{2}}$ add

$$-\frac{4\pi}{2} + \frac{3\pi}{2} = -\frac{\pi}{2}$$

$$\frac{\pi}{2} + \frac{3\pi}{2} = 2\pi$$

P.S.

$$\frac{x}{3} + \frac{\pi}{3} = 0$$

$$3 \cdot \frac{x}{3} = -\frac{\pi}{3} \cdot 3$$

$$x = -\pi$$

$$\frac{4\pi}{2} + \frac{3\pi}{2} = \frac{7\pi}{2}$$

$$\frac{7\pi}{2} + \frac{3\pi}{2} = 5\pi$$

I, I.

$$0 \leq \frac{x}{3} + \frac{\pi}{3} \leq 2\pi$$

$$-\frac{\pi}{3} \quad -\frac{\pi}{3} \quad -\frac{\pi}{3}$$

$$3 \cdot \left(-\frac{\pi}{3}\right) \leq 3 \cdot \left(\frac{x}{3}\right) \leq 3 \cdot \left(5\pi\right)$$

$$-\pi \leq x \leq 5\pi$$

COSINE FUNCTIONS

Standard Form: $y = a \cos(bx - c) + d$

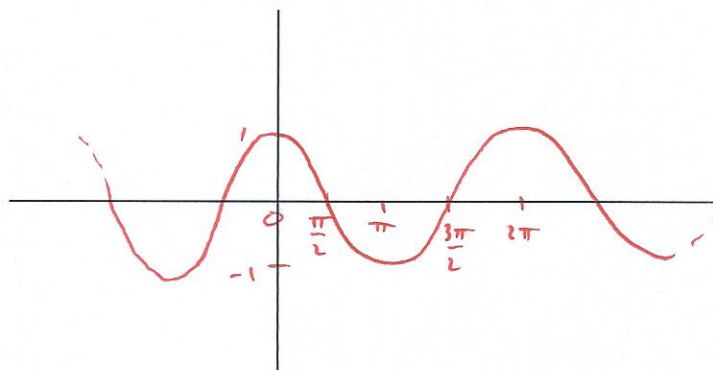
Amplitude: $|a|$

Period: $\frac{2\pi}{|b|}$

Phase Shift: $-\frac{c}{b}$

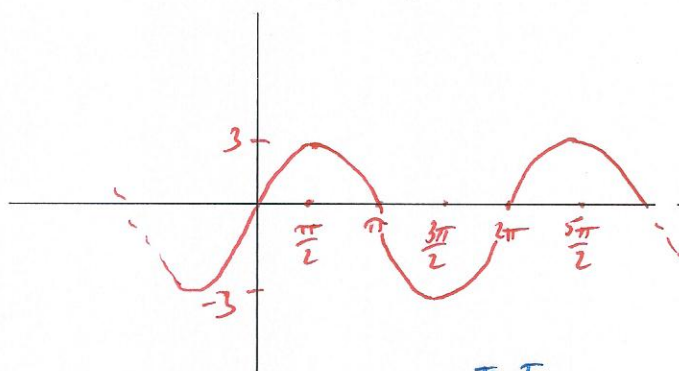
Initial Interval: $0 \leq bx + c \leq 2\pi$

Parent Function $y = \cos x$



Practice Problems

$$y = 3 \cos\left(x - \frac{\pi}{2}\right)$$



Amplitude: 3

Period: 2π

Phase Shift: $\frac{\pi}{2}$

Initial Interval: $\frac{\pi}{2} \leq x \leq \frac{5\pi}{2}$

$$\frac{\text{Period}}{4} = \frac{2\pi}{4} = \boxed{\frac{\pi}{2}} \text{ Add}$$

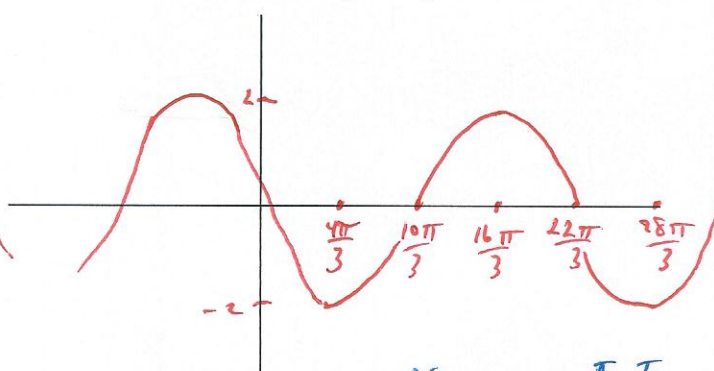
$$\frac{\pi}{2} + \frac{\pi}{2} = \pi$$

$$\frac{2\pi}{2} + \frac{\pi}{2} = \frac{3\pi}{2}$$

$$\frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$$

$$\frac{4\pi}{2} + \frac{\pi}{2} = \frac{5\pi}{2}$$

$$y = -2 \cos\left(\frac{x}{4} - \frac{\pi}{3}\right)$$



Amplitude: 2

Period: $2\pi \div \frac{1}{4} = 8\pi$

Phase Shift: $\frac{4\pi}{3}$

Initial Interval: $\frac{4\pi}{3} \leq x \leq \frac{28\pi}{3}$

$$\frac{\text{Period}}{4} = \frac{8\pi}{4} = \boxed{2\pi} \text{ Add}$$

$$\frac{4\pi}{3} + \frac{6\pi}{3} = \frac{10\pi}{3}$$

$$\frac{10\pi}{3} + \frac{6\pi}{3} = \frac{16\pi}{3}$$

$$\frac{16\pi}{3} + \frac{6\pi}{3} = \frac{22\pi}{3}$$

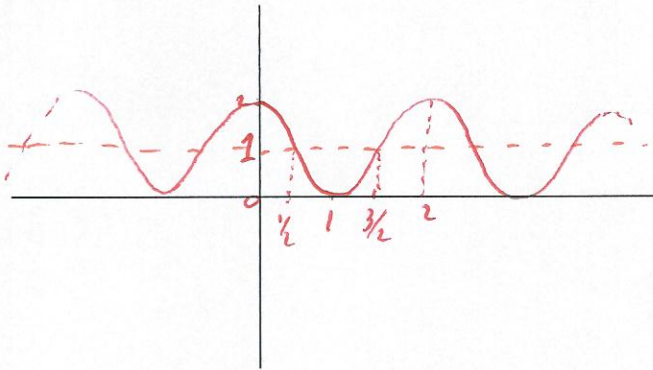
$$\frac{22\pi}{3} + \frac{6\pi}{3} = \frac{28\pi}{3}$$

I.I.
 $0 \leq x - \frac{\pi}{2} \leq 2\pi$
 $+\frac{\pi}{2} \quad +\frac{\pi}{2} \quad +\frac{\pi}{2}$
 $\frac{\pi}{2} \leq x \leq \frac{5\pi}{2}$

P.S.
 $\frac{x}{4} - \frac{\pi}{3} = 0$
 $x = \frac{4\pi}{3}$
I.I.
 $0 \leq \frac{x}{4} - \frac{\pi}{3} \leq 2\pi$
 $+\frac{\pi}{3} \quad +\frac{\pi}{3} \quad +\frac{\pi}{3}$
 $\frac{\pi}{3} \leq \frac{x}{4} \leq \frac{7\pi}{3}$
 $\frac{4\pi}{3} \leq x \leq \frac{28\pi}{3}$

$$y = \cos(\pi x) + 1$$

vertical shift +1



Amplitude: 1

Period: $\frac{2\pi}{\pi} = 2$

Phase Shift: none

Initial Interval: $0 \leq x \leq 2$

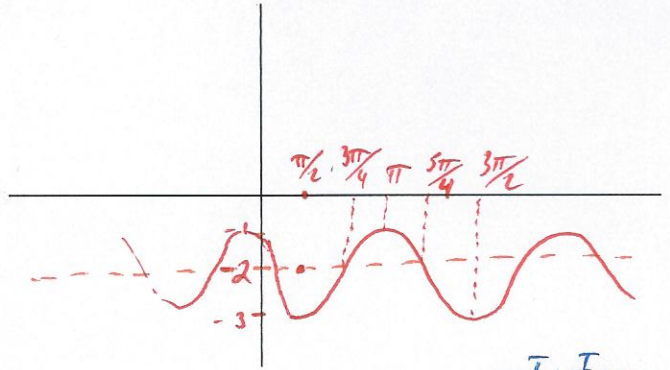
I.I.

$$0 \leq \pi x \leq \frac{2\pi}{\pi}$$

$0 \leq x \leq 2$

$$y = -\cos(2x - \pi) - 2$$

vertical shift -2



Amplitude: 1

Period: $\frac{2\pi}{2} = \pi$

Phase Shift: $\frac{\pi}{2}$

Initial Interval: $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$

period = $\boxed{\frac{\pi}{4}}$ add

I.I.

$$0 \leq 2x - \pi \leq 2\pi$$

$+\pi \quad +\pi \quad +\pi$

$$\pi \leq 2x \leq 3\pi$$

$\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$

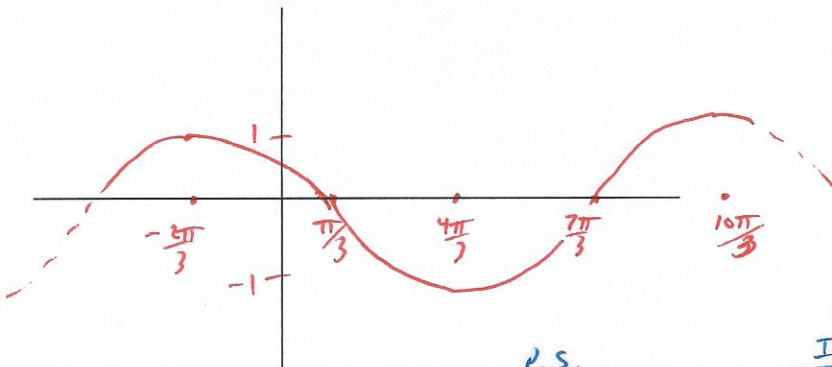
$$\frac{2\pi}{4} + \frac{\pi}{4} = \frac{3\pi}{4}$$

$$\frac{3\pi}{4} + \frac{\pi}{4} = \pi$$

$$\frac{4\pi}{4} + \frac{\pi}{4} = \frac{5\pi}{4}$$

$$\frac{5\pi}{4} + \frac{\pi}{4} = \frac{3\pi}{2}$$

$$y = \cos\left(\frac{x}{2} + \frac{\pi}{3}\right)$$



Amplitude: 1

Period: $2\pi \div \frac{1}{2} = 4\pi$

Phase Shift: $-\frac{2\pi}{3}$

Initial Interval: $-\frac{2\pi}{3} \leq x \leq \frac{10\pi}{3}$

P.S.

$$\frac{x}{2} + \frac{\pi}{3} = 0$$

$$2\left(\frac{x}{2}\right) = 2\left(-\frac{\pi}{3}\right)$$

$$x = -\frac{2\pi}{3}$$

I.I.

$$0 \leq \frac{x}{2} + \frac{\pi}{3} \leq 2\pi$$

$-\frac{\pi}{3} \quad -\frac{\pi}{3} \quad -\frac{\pi}{3}$

$$2\left(-\frac{\pi}{3}\right) \leq 2\left(\frac{x}{2}\right) \leq 2\left(\frac{5\pi}{3}\right)$$

$$-\frac{2\pi}{3} \leq x \leq \frac{10\pi}{3}$$

period = $\frac{4\pi}{4} = \boxed{\pi}$ add

$$-\frac{2\pi}{3} + \frac{3\pi}{3} = \boxed{\frac{\pi}{3}}$$

$$\frac{\pi}{3} + \frac{3\pi}{3} = \boxed{\frac{4\pi}{3}}$$

$$\frac{4\pi}{3} + \frac{3\pi}{3} = \boxed{\frac{7\pi}{3}}$$

$$\frac{7\pi}{3} + \frac{3\pi}{3} = \boxed{\frac{10\pi}{3}}$$

COSECANT FUNCTIONS

Standard Form: $y = a \csc(bx - c) + d \rightarrow$ 1st graph $y = a \sin(bx - c) + d$

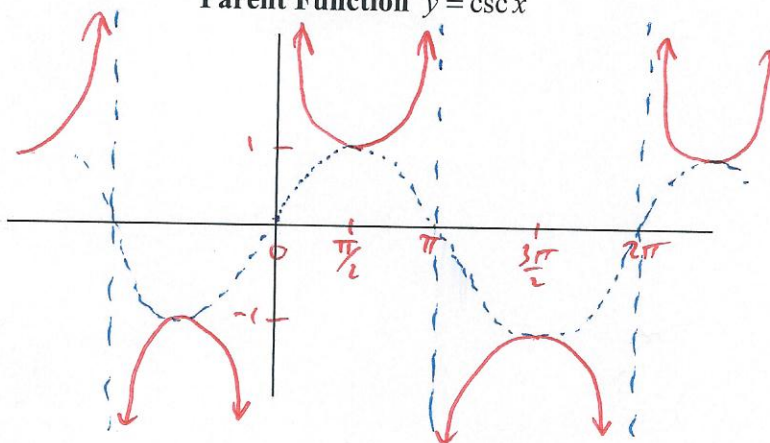
Amplitude: $|a|$

Period: $\frac{2\pi}{|b|}$

Phase Shift: $-\frac{c}{b}$

Initial Interval: $0 \leq bx - c \leq 2\pi$

Parent Function $y = \csc x$



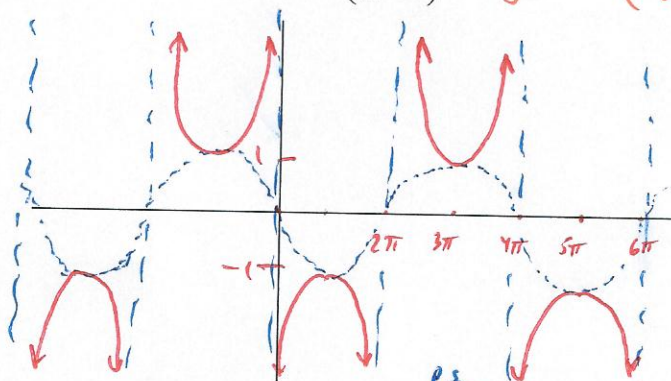
Practice Problems

$$y = \csc\left(\frac{x}{2} - \pi\right)$$

$$y = \sin\left(\frac{x}{2} - \pi\right)$$

$$y = -\csc\left(3x - \frac{\pi}{4}\right)$$

$$y = -\sin\left(3x - \frac{\pi}{4}\right)$$



Amplitude: 1

Period: $2\pi \div \frac{1}{2} = 4\pi$

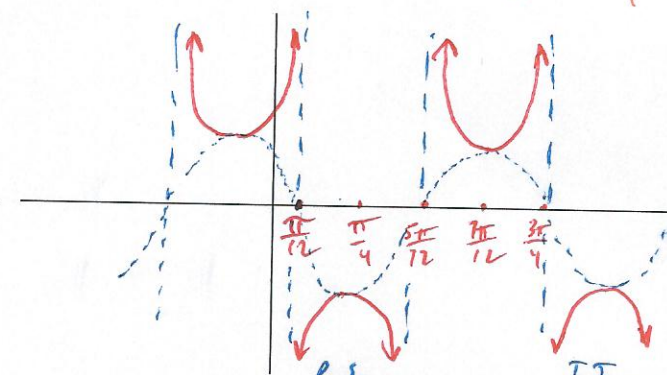
Phase Shift: 2π

Initial Interval: $2\pi \leq x \leq 6\pi$

$$\frac{\text{Period}}{4} = \frac{4\pi}{4} = \boxed{\pi} \text{ Add}$$

P.S.
 $\frac{x}{2} - \pi = 0$
 $\frac{x}{2} = \pi$
 $x = 2\pi$

I.I.
 $0 \leq \frac{x}{2} - \pi \leq 2\pi$
 $+\pi \quad +\pi \quad +\pi$
 $2(\pi) \leq 2\left(\frac{x}{2}\right) \leq 2(3\pi)$
 $2\pi \leq x \leq 6\pi$



Amplitude: 1

Period: $\frac{2\pi}{3}$

Phase Shift: $\pi/12$

Initial Interval: $\pi/12 \leq x \leq 3\pi/4$

$$\frac{\text{Period}}{4} = \frac{2\pi}{3} \div 4 = \frac{2\pi}{3} \cdot \frac{1}{4} = \boxed{\frac{\pi}{6}} \text{ Add}$$

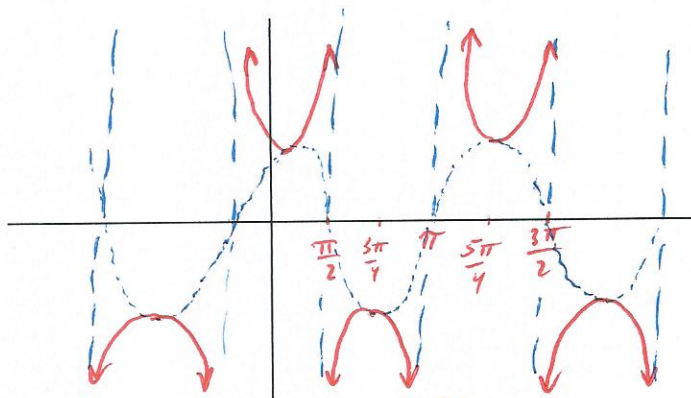
$$\begin{aligned} \frac{\pi}{12} + \frac{2\pi}{12} &= \boxed{\frac{\pi}{4}} \\ \frac{3\pi}{12} + \frac{2\pi}{12} &= \boxed{\frac{5\pi}{12}} \\ \frac{5\pi}{12} + \frac{2\pi}{12} &= \boxed{\frac{7\pi}{12}} \\ \frac{7\pi}{12} + \frac{2\pi}{12} &= \boxed{\frac{3\pi}{4}} \end{aligned}$$

P.S.
 $3x - \frac{\pi}{4} = 0$
 $3x = \frac{\pi}{4} \cdot \frac{1}{3}$
 $x = \frac{\pi}{12}$

I.I.
 $0 \leq 3x - \frac{\pi}{4} \leq 2\pi$
 $+\frac{\pi}{4} \quad +\frac{\pi}{4} \quad +\frac{\pi}{4}$
 $\frac{\pi}{4} \leq 3x \leq 2\pi + \frac{\pi}{4}$
 $\frac{1}{3}\left(\frac{\pi}{4}\right) \leq \frac{3x}{3} \leq \frac{1}{3}\left(2\pi + \frac{\pi}{4}\right)$
 $\frac{\pi}{12} \leq x \leq \frac{3\pi}{4}$

$$y = 2 \csc(\pi - 2x) \quad y = -2 \sin(2x - \pi)$$

$$y = -\csc\left(x + \frac{\pi}{4}\right) + 1 \quad y = -\sin\left(x + \frac{\pi}{4}\right) + 1$$



I.I.

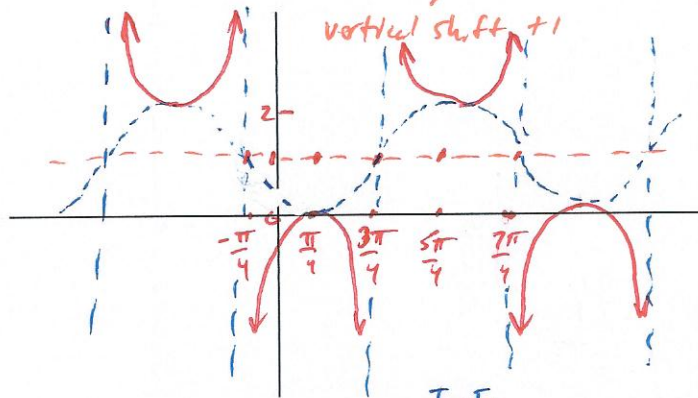
Amplitude: 2

Period: $\frac{2\pi}{2} = \pi$

Phase Shift: $\frac{\pi}{2}$

Initial Interval: $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$

$$\begin{aligned} \frac{\text{period}}{4} &= \left\lfloor \frac{\pi}{4} \right\rfloor \text{ Add} \\ \frac{2\pi}{4} + \frac{\pi}{4} &= \left\lfloor \frac{3\pi}{4} \right\rfloor \\ \frac{3\pi}{4} + \frac{\pi}{4} &= \left\lfloor \pi \right\rfloor \\ \frac{4\pi}{4} + \frac{\pi}{4} &= \left\lfloor \frac{5\pi}{4} \right\rfloor \\ \frac{5\pi}{4} + \frac{\pi}{4} &= \left\lfloor \frac{3\pi}{2} \right\rfloor \end{aligned}$$



I.I.

Amplitude: 1

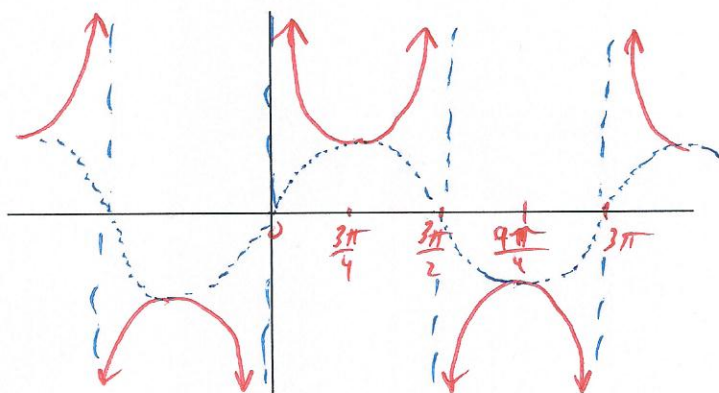
Period: 2π

Phase Shift: $-\frac{\pi}{4}$

Initial Interval: $-\frac{\pi}{4} \leq x \leq \frac{7\pi}{4}$

$$\begin{aligned} \frac{\text{period}}{4} &= \frac{2\pi}{4} = \left\lfloor \frac{\pi}{2} \right\rfloor \text{ Add} \\ -\frac{\pi}{4} + \frac{2\pi}{4} &= \left\lfloor \frac{\pi}{4} \right\rfloor \\ \frac{\pi}{4} + \frac{2\pi}{4} &= \left\lfloor \frac{3\pi}{4} \right\rfloor \\ \frac{3\pi}{4} + \frac{2\pi}{4} &= \left\lfloor \frac{5\pi}{4} \right\rfloor \\ \frac{5\pi}{4} + \frac{2\pi}{4} &= \left\lfloor \frac{7\pi}{4} \right\rfloor \end{aligned}$$

$$y = \csc\left(\frac{2x}{3}\right) \quad y = \sin\left(\frac{2x}{3}\right)$$



Amplitude: 1

Period: $2\pi \div \frac{2}{3} = 2\pi \cdot \frac{3}{2} = 3\pi$

Phase Shift: none

Initial Interval: $0 \leq x \leq 3\pi$

$$\begin{aligned} \frac{\text{period}}{4} &= \left\lfloor \frac{3\pi}{4} \right\rfloor \text{ Add} \\ 0 + \frac{3\pi}{4} &= \left\lfloor \frac{3\pi}{4} \right\rfloor \\ \frac{3\pi}{4} + \frac{3\pi}{4} &= \left\lfloor \frac{3\pi}{2} \right\rfloor \end{aligned}$$

$$\begin{aligned} \frac{6\pi}{4} + \frac{3\pi}{4} &= \left\lfloor \frac{9\pi}{4} \right\rfloor \\ \frac{9\pi}{4} + \frac{3\pi}{4} &= \left\lfloor 3\pi \right\rfloor \end{aligned}$$

SECANT FUNCTIONS

Standard Form: $y = a \sec(bx - c) + d$

1st graph $y = a \cos(bx - c) + d$

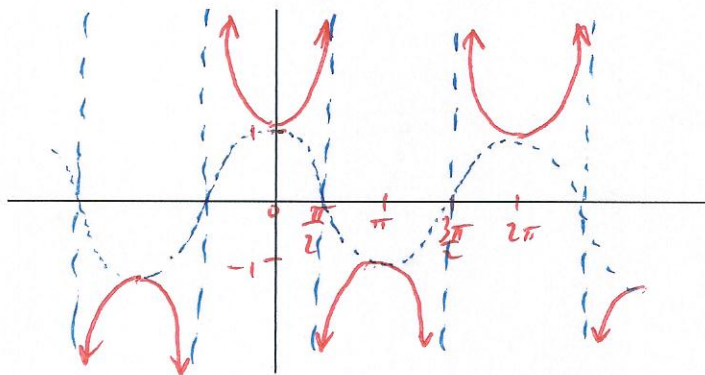
Amplitude: $|a|$

Period: $\frac{2\pi}{|b|}$

Phase Shift: $-\frac{c}{b}$

Initial Interval: $0 \leq bx - c \leq 2\pi$

Parent Function $y = \sec x$



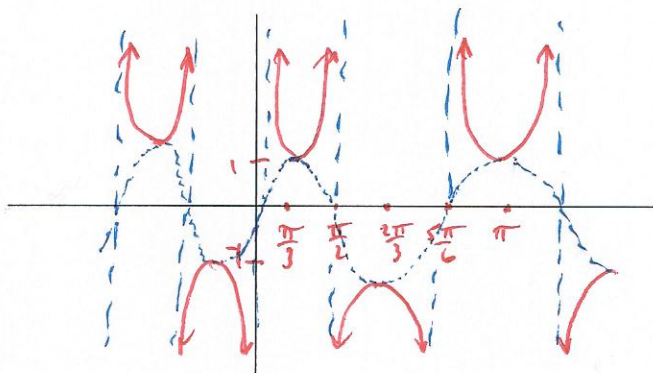
Practice Problems

$$y = \sec(3x - \pi)$$

$$y = \cos(3x - \pi)$$

$$y = -2 \sec\left(x - \frac{\pi}{3}\right)$$

$$y = -2 \cos\left(x - \frac{\pi}{3}\right)$$



Amplitude: 1

Period: $\frac{2\pi}{3}$

Phase Shift: $\frac{\pi}{3}$

Initial Interval: $\frac{\pi}{3} \leq x \leq \pi$

$$0 \leq 3x - \pi \leq 2\pi$$

$$+\pi \quad +\pi \quad +\pi$$

$$\frac{\pi}{3} \leq 3x \leq \frac{5\pi}{3}$$

$$\frac{\pi}{3} \leq x \leq \pi$$

$$\frac{\pi}{3} \leq x \leq \pi$$

$$\frac{\pi}{3} \leq x \leq \pi$$

Amplitude: 2

Period: 2π

Phase Shift: $\frac{\pi}{3}$

Initial Interval: $\frac{\pi}{3} \leq x \leq \frac{7\pi}{3}$

$$0 \leq x - \frac{\pi}{3} \leq 2\pi$$

$$+\pi \quad +\pi \quad +\pi$$

$$\frac{\pi}{3} \leq x \leq \frac{7\pi}{3}$$

$$\frac{\pi}{3} \leq x \leq \frac{7\pi}{3}$$

$$\frac{\pi}{3} \leq x \leq \frac{7\pi}{3}$$

$$\frac{\pi}{3} \leq x \leq \frac{7\pi}{3}$$

$$\frac{\text{Period}}{4} = \frac{2\pi}{3} \cdot \frac{1}{4} = \boxed{\frac{\pi}{6}} \text{ Add}$$

$$\frac{2\pi}{6} + \frac{\pi}{6} = \boxed{\frac{\pi}{2}}$$

$$\frac{3\pi}{6} + \frac{\pi}{6} = \boxed{\frac{2\pi}{3}}$$

$$\frac{4\pi}{6} + \frac{\pi}{6} = \boxed{\frac{5\pi}{6}}$$

$$\frac{5\pi}{6} + \frac{\pi}{6} = \boxed{\pi}$$

$$\frac{\text{Period}}{4} = \frac{2\pi}{4} = \boxed{\frac{\pi}{2}} \text{ Add}$$

$$\frac{\pi}{3} + \frac{\pi}{2} = \frac{2\pi}{6} + \frac{3\pi}{6} = \boxed{\frac{5\pi}{6}}$$

$$\frac{5\pi}{6} + \frac{\pi}{6} = \frac{6\pi}{6} = \boxed{\pi}$$

$$\frac{8\pi}{6} + \frac{\pi}{6} = \boxed{\frac{9\pi}{6}}$$

$$\frac{11\pi}{6} + \frac{\pi}{6} = \frac{12\pi}{6} = \boxed{2\pi}$$

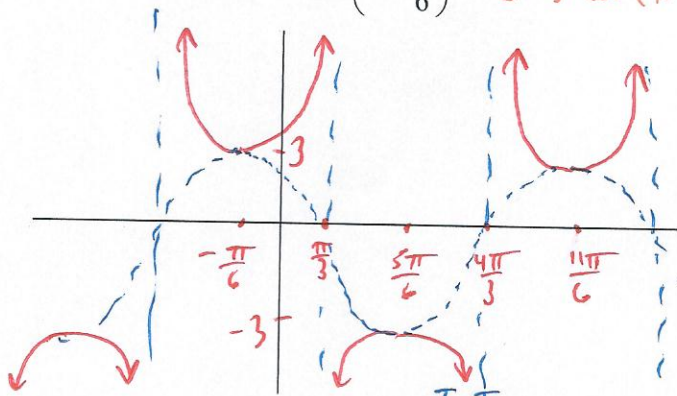
$$y = 3 \sec\left(x + \frac{\pi}{6}\right)$$

$$y = 3 \cos\left(x + \frac{\pi}{6}\right)$$

vertical shift +1

$$y = -2 \sec\left(3x - \frac{\pi}{2}\right) + 1$$

$$y = -2 \cos\left(3x - \frac{\pi}{2}\right) + 1$$



Amplitude: 3

Period: 2π

Phase Shift: $-\frac{\pi}{6}$

Initial Interval: $-\frac{\pi}{6} \leq x \leq \frac{11\pi}{6}$

$$\frac{\text{Period}}{4} = \frac{2\pi}{4} = \left\lfloor \frac{\pi}{2} \right\rfloor \text{ Add}$$

$$-\frac{\pi}{6} + \frac{3\pi}{2} = \left\lfloor \frac{\pi}{3} \right\rfloor$$

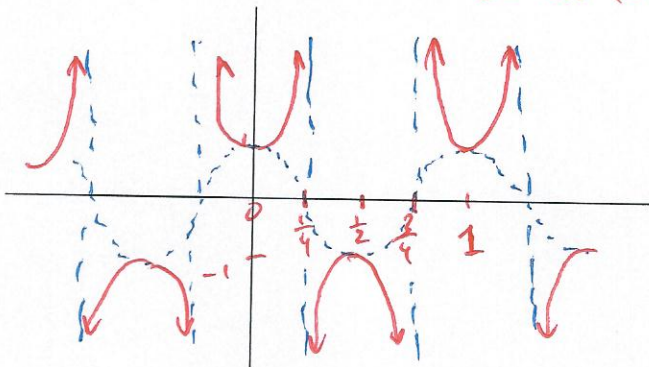
$$\frac{2\pi}{6} + \frac{3\pi}{2} = \left\lfloor \frac{5\pi}{6} \right\rfloor$$

$$\frac{3\pi}{6} + \frac{3\pi}{2} = \left\lfloor \frac{4\pi}{3} \right\rfloor$$

$$\frac{8\pi}{6} + \frac{3\pi}{2} = \left\lfloor \frac{11\pi}{6} \right\rfloor$$

$$y = \sec(2\pi x)$$

$$y = \cos(2\pi x)$$



Amplitude: 1

Period: $\frac{2\pi}{2\pi} = 1$

Phase Shift: none

Initial Interval: $0 \leq x \leq 1$

$$\frac{\text{Period}}{4} = \left\lfloor \frac{1}{4} \right\rfloor$$

Amplitude: 2

Period: $\frac{2\pi}{3}$

Phase Shift: $\frac{\pi}{6}$

Initial Interval: $\frac{\pi}{6} \leq x \leq \frac{5\pi}{6}$

$$\frac{\text{Period}}{4} = \frac{2\pi}{3} \cdot \frac{1}{4} = \left\lfloor \frac{\pi}{6} \right\rfloor \text{ Add}$$

$$\frac{\pi}{6} + \frac{\pi}{6} = \left\lfloor \frac{\pi}{3} \right\rfloor$$

$$\frac{2\pi}{6} + \frac{\pi}{6} = \left\lfloor \frac{\pi}{2} \right\rfloor$$

$$\frac{3\pi}{6} + \frac{\pi}{6} = \left\lfloor \frac{2\pi}{3} \right\rfloor$$

$$\frac{4\pi}{6} + \frac{\pi}{6} = \left\lfloor \frac{5\pi}{6} \right\rfloor$$

P.S.

$$3x - \frac{\pi}{2} = 0$$

$$\left(\frac{1}{3}\right)3x = \frac{\pi}{2} \left(\frac{1}{3}\right)$$

$$x = \frac{\pi}{6}$$

I.I.

$$0 \leq 3x - \frac{\pi}{2} \leq 2\pi$$

$$\frac{\pi}{2} \leq 3x \leq \frac{5\pi}{2}$$

$$\left(\frac{1}{3}\right)\frac{\pi}{2} \leq \frac{3x}{3} \leq \frac{5\pi}{3}$$

$$\frac{\pi}{6} \leq x \leq \frac{5\pi}{6}$$

TANGENT FUNCTIONS

Standard Form: $y = a \tan(bx - c) + d$

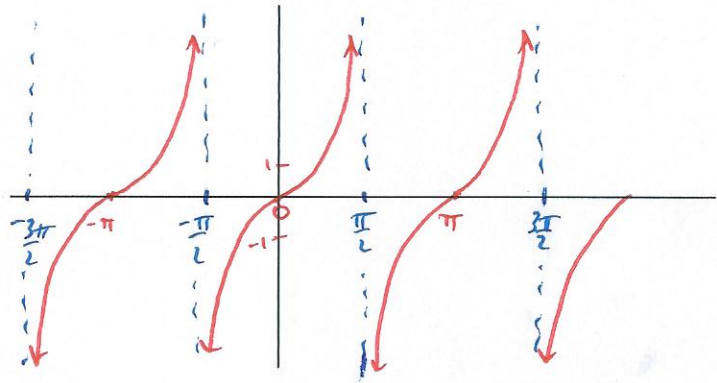
Amplitude: $|a|$

Period: $\frac{\pi}{|b|}$

Phase Shift:

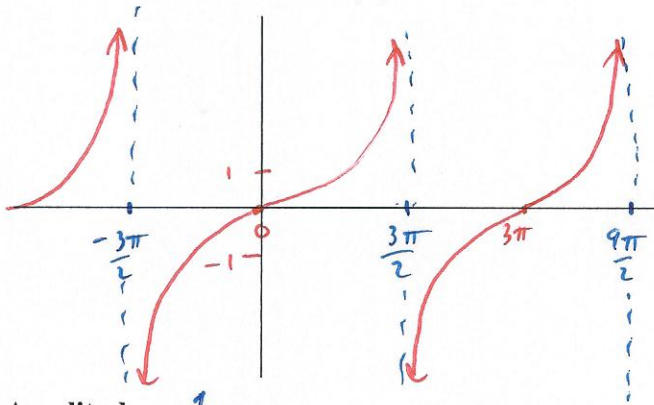
Initial Interval: $-\frac{\pi}{2} < bx - c < \frac{\pi}{2}$

Parent Function $y = \tan x$



Practice Problems

$$y = \tan\left(\frac{x}{3}\right)$$



Amplitude: 1

Period: $\pi \div \frac{1}{3} = 3\pi$

Phase Shift:

Initial Interval: $-\frac{3\pi}{2} < x < \frac{3\pi}{2}$

$$\left\{ \left(-\frac{\pi}{2}\right) < \left(\frac{x}{3}\right) < \left(\frac{\pi}{2}\right) \right\}$$

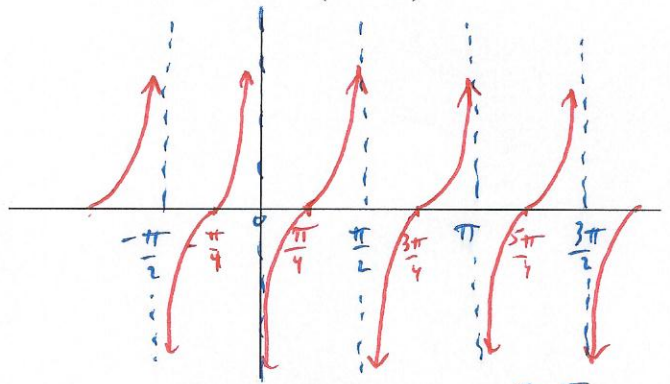
$$-\frac{3\pi}{2} < x < \frac{3\pi}{2}$$

$$\boxed{\frac{3\pi}{2}} \text{ add}$$

$$\frac{3\pi}{2} + \frac{3\pi}{2} = \frac{6\pi}{2}$$

$$\frac{3\pi}{2} + \frac{6\pi}{2} = \frac{9\pi}{2}$$

$$y = \tan\left(2x - \frac{\pi}{2}\right)$$



Amplitude: 1

Period: $\frac{\pi}{2}$

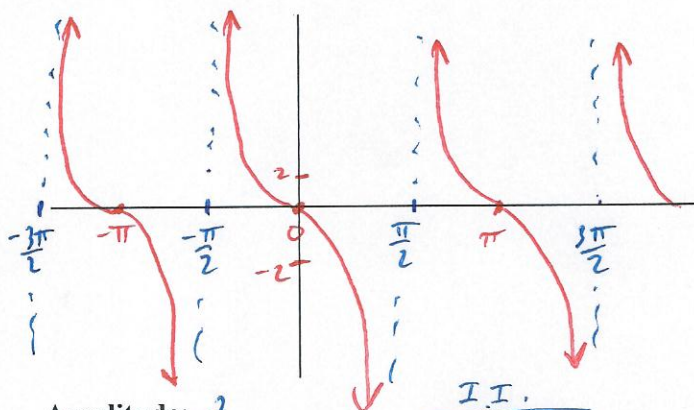
Phase Shift:

Initial Interval: $0 < x < \frac{\pi}{2}$

I. I.

$$\begin{array}{r} -\frac{\pi}{2} < 2x - \frac{\pi}{2} < \frac{\pi}{2} \\ +\frac{\pi}{2} \quad +\frac{\pi}{2} \quad +\frac{\pi}{2} \\ \hline 0 < 2x < \pi \\ \hline 0 < x < \frac{\pi}{2} \end{array}$$

$$y = -2 \tan(x + \pi)$$



Amplitude: 2

Period: π

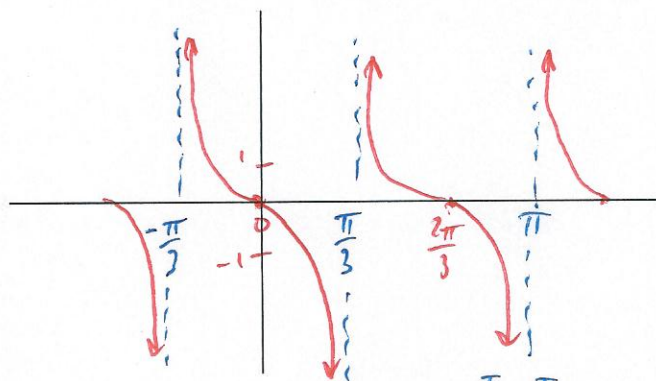
Phase Shift: -

Initial Interval: $-\frac{3\pi}{2} < x < -\frac{\pi}{2}$

$$\frac{\text{period}}{2} = \left[\frac{\pi}{2} \right] \text{ sec}$$

$$-\frac{3\pi}{2} + \frac{\pi}{2} = -\frac{2\pi}{2}$$

$$y = -\tan\left(\frac{3x}{2}\right)$$



Amplitude: 1

Period: $\pi \div \frac{3}{2} = \frac{2\pi}{3}$

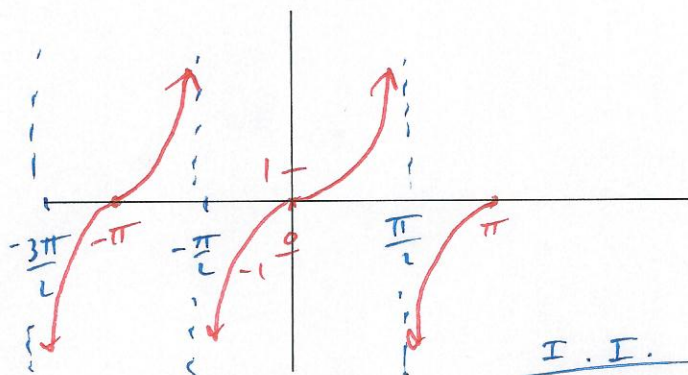
Phase Shift: -

Initial Interval: $-\frac{\pi}{3} < x < \frac{\pi}{3}$

$$\frac{\text{period}}{2} = \frac{2\pi}{3} \cdot \frac{1}{2} = \left[\frac{\pi}{3} \right] \quad \frac{\pi}{3} + \frac{2\pi}{3} = \pi$$

$$\frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$$

$$y = \tan(x + \pi)$$



Amplitude: 1

Period: π

Phase Shift: -

Initial Interval: $-\frac{3\pi}{2} < x < -\frac{\pi}{2}$

$$\begin{aligned} & \text{I.I.} \\ & -\frac{\pi}{2} < x + \pi < \frac{\pi}{2} \\ & -\pi & -\pi & -\pi \\ & -\frac{3\pi}{2} < x < -\frac{\pi}{2} \end{aligned}$$

COTANGENT FUNCTIONS

Standard Form: $y = a \cot(bx - c) + d$

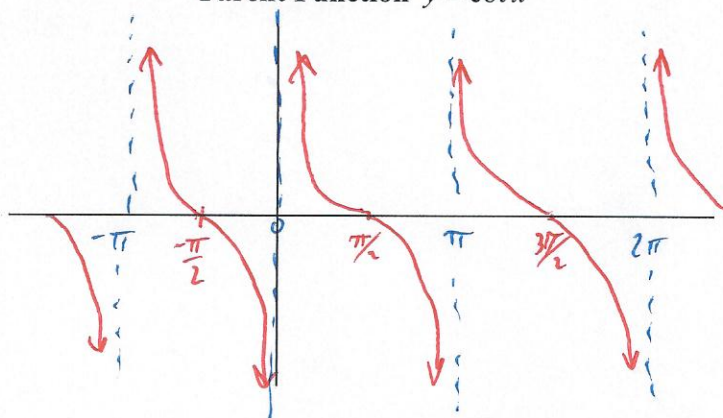
Amplitude: $|a|$

Period: $\frac{\pi}{|b|}$

Phase Shift: $-$

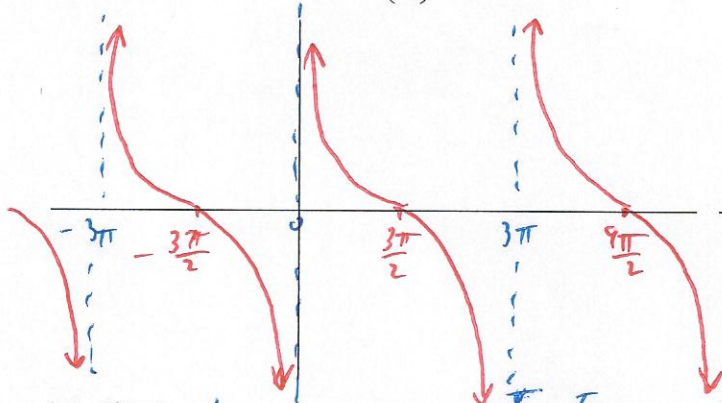
Initial Interval: $0 < bx - c < \pi$

Parent Function $y = \cot x$



Practice Problems

$$y = \cot\left(\frac{x}{3}\right)$$



Amplitude: 1

Period: $\pi \div \frac{1}{3} = 3\pi$

Phase Shift: $-$

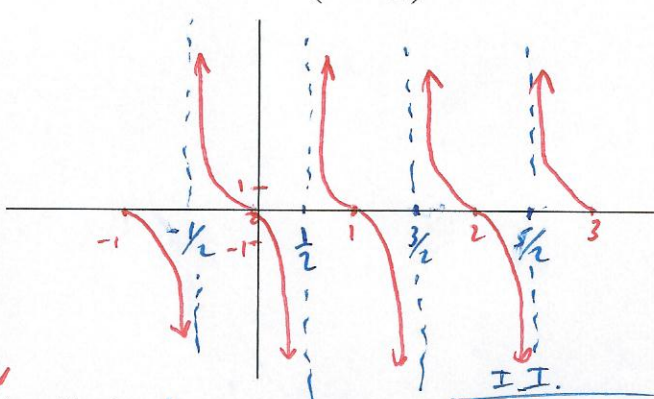
Initial Interval: $0 < x < 3\pi$

$$\frac{\text{period}}{2} = \boxed{\frac{3\pi}{2}} \text{ Add}$$

$$3\pi + \frac{3\pi}{2} = \frac{9\pi}{2}$$

$$\frac{6\pi}{2} + \frac{3\pi}{2}$$

$$y = \cot\left(\pi x - \frac{\pi}{2}\right)$$



Amplitude: 1

Period: $\frac{\pi}{\pi} = 1$

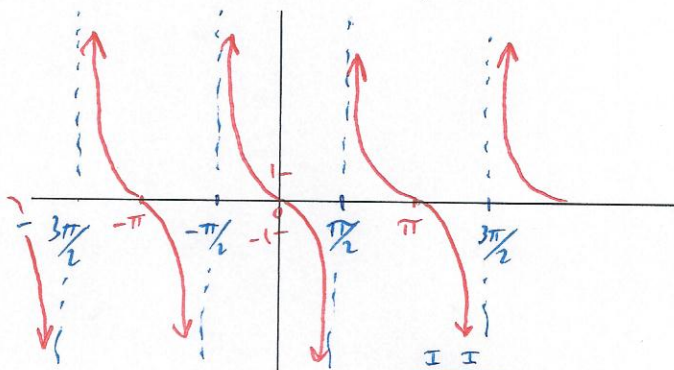
Phase Shift: $-$

Initial Interval: $\frac{1}{2} < x < \frac{3}{2}$

$$\frac{\text{period}}{2} = \boxed{\frac{1}{2}} \text{ Add}$$

$$\frac{1}{2} < x < \frac{3}{2}$$

$$y = \cot\left(x - \frac{\pi}{2}\right)$$



Amplitude: 1

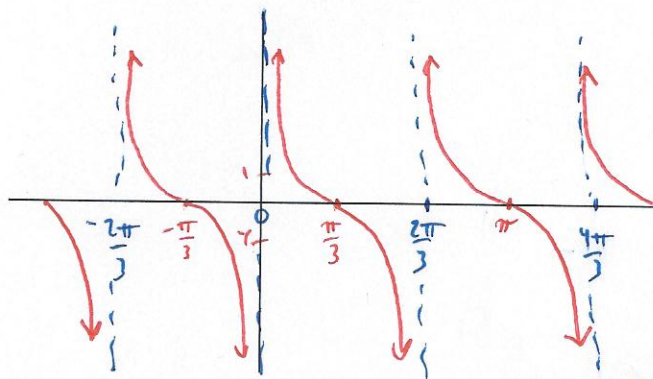
Period: π

Phase Shift: -

Initial Interval: $\frac{\pi}{2} < x < \frac{3\pi}{2}$

$$\begin{array}{c} \text{I I} \\ 0 < x - \frac{\pi}{2} < \pi \\ +\frac{\pi}{2} \quad +\frac{\pi}{2} \quad +\frac{\pi}{2} \\ \frac{\pi}{2} < x < \frac{3\pi}{2} \end{array}$$

$$y = \cot\left(\frac{3x}{2} - \pi\right)$$



Amplitude: 1

Period: $\pi \div \frac{3}{2} = \frac{2\pi}{3}$

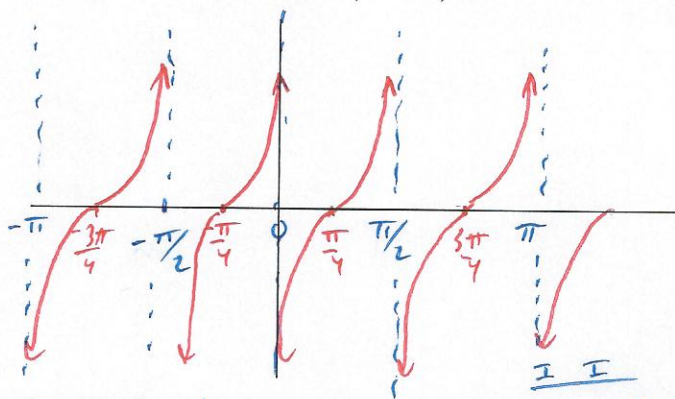
Phase Shift: -

Initial Interval: $\frac{2\pi}{3} < x < \frac{4\pi}{3}$

$$\begin{array}{c} \text{I I} \\ 0 < \frac{3x}{2} - \pi < \pi \\ +\pi \quad +\pi \quad +\pi \\ \frac{2}{3}(\pi) < \frac{2}{3}\left(\frac{3x}{2}\right) < \frac{2}{3}(2\pi) \end{array}$$

$$\begin{array}{l} \frac{2\pi}{3} \div 2 \\ \frac{2\pi}{3} \cdot \frac{1}{2} = \boxed{\frac{\pi}{3}} \end{array} \quad \text{Add}$$

$$y = -\cot(2x + \pi)$$



Amplitude: 1

Period: $\frac{\pi}{2}$

Phase Shift: -

Initial Interval: $-\frac{\pi}{2} < x < 0$

$$\begin{array}{c} \text{I I} \\ 0 < 2x + \pi < \pi \\ -\pi \quad -\pi \quad -\pi \\ -\frac{\pi}{2} < \frac{2x}{2} < \frac{0}{2} \\ -\frac{\pi}{2} < x < 0 \end{array}$$

$$\frac{\text{period}}{2} = \frac{\pi}{2} \div 2 = \frac{\pi}{2} \cdot \frac{1}{2} = \boxed{\frac{\pi}{4}} \quad \text{Add}$$