

CHINO VALLEY UNIFIED SCHOOL DISTRICT
INSTRUCTIONAL GUIDE
TRIGONOMETRY / PRE-CALCULUS

Course Number	5121
Department	Mathematics
Qualification Guidelines	Successful completion of both semesters of Algebra 1 or Algebra 1 Academy, Geometry, <u>and</u> Algebra II (30 credits)
Length of Course	Two (2) semesters/One (1) year
Grade Level	10-12
Credit	5 credits per semester/10 total credits - mathematics
Repeatable	Not repeatable for credit
UC/CSU	Meets "c" mathematics requirement
Board Approved	June 19, 2008

Description of Course – Trigonometry combines many of the algebraic, geometric, and trigonometric techniques needed to prepare students for the study of calculus and strengthens their understanding of problems and mathematical reasoning in solving problems. This course takes a functional approach towards those topics. Students learn the techniques of matrix manipulation so they can solve systems of linear equations in any number of variables. The trigonometry functions studied are defined geometrically, rather than in terms of algebraic equations. Students must understand the concepts of trigonometric functions and have the ability to prove basic trigonometric identities. This course incorporates a combination of the California State Content Standards in Mathematical Analysis, Linear Algebra, Trigonometry, and Probability and Statistics.

Rationale for Course – Knowledge of trigonometric functions as well as being able to prove basic trigonometric identities regarding these functions is especially important for students intending to study calculus, more advanced mathematics, physics and other sciences, and engineering in college. The standards included in this course from the Mathematical Analysis and Linear Algebra courses combine with the standards from the Trigonometry to create a more robust Trigonometry/Pre-Calculus course to prepare students for the study of statistics and/or calculus and college or university entrance tests. Three California Content Standards in Probability and Statistics included on the Summative High School Mathematics California Standards Test have been included in this curriculum guide for Trigonometry/Pre-Calculus, so that students are properly prepared for these assessment questions.

Trigonometry Standards

Standard 1 - Students understand the notion of angle and how to measure it, in both degrees and radians. They can convert between degrees and radians.

1.1 Objective: Knowledge of angle vocabulary.

1.1.1 Performance Indicator: Given a problem utilizing trigonometric vocabulary, the student will have an understanding of the vocabulary including: angle, initial side, terminal side, vertex, standard position, quadrant, positive and negative angles, coterminal angles, degrees, minutes, seconds, radians, supplementary angles, and complementary angles.

1.2 Objective: Calculating angle values.

1.2.1 Performance Indicator: Given an angle in standard position on the unit circle, the student will be able to express its measure in degrees or radians.

1.2.2 Performance Indicator: Given the measure of an angle in degrees (radians), the student will be able to express its measure in radians (degrees).

1.2.3 Performance Indicator: Given the measure of an angle in decimal degrees (degrees, minutes, seconds), the student will be able to express its measure in degrees, minutes, seconds (decimal degrees).

1.2.4 Performance Indicator: Given an angle in standard position on the unit circle, the student will be able to express its measure and name one positive and one negative coterminal angle in degrees or radians.

1.2.5 Performance Indicator: Given an angle in standard position on the unit circle, the student will be able to name a reference angle in the first quadrant.

Standard 2 - Students know the definition of sine and cosine as y - and x - coordinates of points on the unit circle and are familiar with the graphs of the sine and cosine functions.

2.1 Objective: Knowledge of sine and cosine function values.

2.1.1 Performance Indicator: Given an angle (θ) in standard position, with coordinates (x,y) on its terminal side, the student will be able to determine the following values: $\cos \theta = \frac{x}{r}$ and $\sin \theta = \frac{y}{r}$.

- 2.1.2 Performance Indicator: Given the unit circle, the student will be able to determine that any angle (θ), with coordinates (x,y) on its terminal side, has the following values: $\cos\theta = x$ and $\sin\theta = y$.
- 2.1.3 Performance Indicator: Given an angle (θ), whose measure is a positive or negative multiple of 30° ($\frac{\pi}{6}$) or 45° ($\frac{\pi}{4}$), the student will be able to determine $\sin\theta$ and $\cos\theta$.
- 2.2 Objective: Knowledge of the graphs of the sine and cosine functions.
- 2.2.1 Performance Indicator: The student will be able to graph $y = \sin x$ and $y = \cos x$.
- 2.2.2 Performance Indicator: Given a sketch of either a sine or cosine functions, the student will be able to identify the function.

Standard 3 - Students know the identity $\cos^2 x + \sin^2 x = 1$:

- 3.1 Objective: Students prove that this identity is equivalent to the Pythagorean Theorem (i.e., students can prove this identity by using the Pythagorean Theorem and, conversely, they can prove the Pythagorean Theorem as a consequence of this identity).
- 3.1.1 Performance Indicator: Given a coordinate on the terminal side of an angle in standard position, the student will be able to draw a right triangle back to the x-axis and use the definitions of sine and cosine of the angle to prove the trigonometric Pythagorean identity, $\cos^2 x + \sin^2 x = 1$.
- 3.1.2 Performance Indicator: Given either the sine value or cosine value of an angle, the student will be able to calculate the other trigonometric value using the Pythagorean identity.
- 3.2 Objective: Students prove other trigonometric identities and simplify others by using the identity $\cos^2 x + \sin^2 x = 1$. For example, students use this identity to prove that $\sec^2 x = \tan^2 x + 1$.
- 3.2.1 Performance Indicator: Given the trigonometric Pythagorean identity $\cos^2 x + \sin^2 x = 1$, the student will be able to divide each term by $\cos^2 x$ to prove that $\sec^2 x = \tan^2 x + 1$.
- 3.2.2 Performance Indicator: Given either the tangent value or secant value of an angle, the student will be able to calculate the other trigonometric value using the Pythagorean identity.

3.2.3 Performance Indicator: Given the trigonometric Pythagorean identity $\cos^2 x + \sin^2 x = 1$, the student will be able to divide each term by $\sin^2 x$ to prove that $\cot^2(x) + 1 = \csc^2(x)$.

3.2.4 Performance Indicator: Given either the cotangent value or cosecant value of an angle, the student will be able to calculate the other trigonometric value using the Pythagorean identity.

Standard 4 - Students graph functions of the form $f(t) = A \sin(Bt + C)$ or $f(t) = A \cos(Bt + C)$ and interpret A , B , and C in terms of amplitude, frequency, period, and phase shift.

4.1 Objective: Knowledge of the effect of a coefficient on the graph of sine and cosine function.

4.1.1 Performance Indicator: Given an equation of the form $y = A \sin(Bx + C)$ or $y = A \cos(Bx + C)$, the student will be able to graph the equation.

4.1.2 Performance Indicator: Given an equation of the form $y = A \sin(Bx + C)$ or $y = A \cos(Bx + C)$, the student will be able to determine the function's period, amplitude, and phase shift.

4.1.3 Performance Indicator: Given a sketch of the sine or cosine function of the form $y = A \sin(Bx + C)$ or $y = A \cos(Bx + C)$, the student will be able to write the equation matching the graph.

Standard 5 - Students know the definitions of the tangent and cotangent functions and can graph them.

5.1 Objective: Knowledge of tangent and cotangent function values.

5.1.1 Performance Indicator: Given an angle (θ) in standard position, with coordinates (x, y) on its terminal side, the student will be able to determine the following values: $\tan \theta = \frac{y}{x}$ and $\cot \theta = \frac{x}{y}$.

5.1.2 Performance Indicator: Given an angle (θ), whose measure is a positive or negative multiple of 30° ($\frac{\pi}{6}$) or 45° ($\frac{\pi}{4}$), the student will be able to determine $\tan \theta$ and $\cot \theta$.

5.1.3 Performance Indicator: Given the unit circle, the student will be able to determine that any angle (θ), with coordinates (x,y) on its terminal side, has the following values: $\tan \theta = \frac{y}{x}$ and $\cot \theta = \frac{x}{y}$.

5.1.4 Performance Indicator: Given the sine and cosine of an angle (θ), the student will be able to determine the tangent and cotangent using the quotient identities: $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and $\cot \theta = \frac{\cos \theta}{\sin \theta}$.

5.2 Objective: Knowledge of the graphs of the tangent and cotangent functions.

5.2.1 Performance Indicator: Given a sketch of either a tangent or cotangent function, the student will be able to identify the function.

5.2.2 Performance Indicator: The student will be able to graph $y = \tan x$ and $y = \cot x$.

5.2.3 Performance Indicator: Given an equation of the form $y = A \tan(Bx + C)$ or $y = A \cot(Bx + C)$, the student will be able to graph the equation and determine the period and phase shift.

5.2.4 Performance Indicator: Given a tangent or cotangent function, the student will be able to determine where the function is undefined, and whether the function is increasing or decreasing.

Standard 6 - Students know the definitions of the secant and cosecant functions and can graph them.

6.1 Objective: Knowledge of secant and cosecant function values.

6.1.1 Performance Indicator: Given an angle (θ) in standard position, with coordinates (x,y) on its terminal side, the student will be able to determine the following values: $\sec \theta = \frac{r}{x}$ and $\csc \theta = \frac{r}{y}$.

6.1.2 Performance Indicator: Given an angle (θ), whose measure is a positive or negative multiple of 30° ($\frac{\pi}{6}$) or 45° ($\frac{\pi}{4}$), the student will be able to determine $\sec \theta$ and $\csc \theta$.

6.1.3 Performance Indicator: Given the unit circle, the student will be able to determine that any angle (θ), with coordinates (x,y) on its terminal side, has the following values: $\sec \theta = \frac{1}{x}$ and $\csc \theta = \frac{1}{y}$.

6.1.4 Performance Indicator: Given the sine and cosine of an angle (θ), the student will be able to determine the secant and cosecant using the quotient identities: $\sec \theta = \frac{1}{\cos \theta}$ and $\csc \theta = \frac{1}{\sin \theta}$.

6.2 Objective: Knowledge of the graphs of the secant and cosecant functions.

6.2.1 Performance Indicator: Given a sketch of either a secant or cosecant function, the student will be able to identify the function.

6.2.2 Performance Indicator: The student will be able to graph $y = \sec x$ and $y = \csc x$.

6.2.3 Performance Indicator: Given an equation of the form $y = A \sec(Bx + C)$ or $y = A \csc(Bx + C)$, the student will be able to graph the equation.

6.2.4 Performance Indicator: Given a secant or cosecant function, the student will be able to determine where the function is undefined.

Standard 7 - Students know that the tangent of the angle that a line makes with the x-axis is equal to the slope of the line.

7.1 Objective: Knowledge of angles of lines and slope of lines.

7.1.1 Performance Indicator: Given an equation of a line, the student will be able to prove that the slope of the line is the tangent of the angle the line makes with the x-axis.

7.1.2 Performance Indicator: Given two points, the student will be able to calculate the slope of the line containing them and determine the angle the line makes with the x-axis.

Standard 8 - Students know the definitions of the inverse trigonometric functions and can graph the functions.

8.1 Objective: Knowledge of the inverse functions.

8.1.1 Performance Indicator: Given any of the three inverse trigonometric functions, the student will be able to define the inverse functions as:

- $y = \arcsin x$ if and only if $\sin y = x$
- $y = \arccos x$ if and only if $\cos y = x$
- $y = \arctan x$ if and only if $\tan y = x$

8.1.2 Performance Indicator: Given any of the three inverse trigonometric functions, the student will be able to define the domain of the inverse function as:

- $y = \arcsin x$ Domain: $-1 \leq x \leq 1$
- $y = \arccos x$ Domain: $-1 \leq x \leq 1$
- $y = \arctan x$ Domain: $-\infty \leq x \leq \infty$

Standard 9 - Students compute, by hand, the values of the trigonometric functions and the inverse trigonometric functions at various standard points.

9.1 Objective: Knowledge of the values of trigonometric and the inverse trigonometric functions.

9.1.1 Performance Indicator: Given an angle (θ), whose measure is a positive or negative multiple of 30° ($\frac{\pi}{6}$) or 45° ($\frac{\pi}{4}$), the student will be able to determine $\sin \theta$, $\cos \theta$, $\tan \theta$, $\cot \theta$, $\sec \theta$, $\csc \theta$ without the use of a calculator.

9.1.2 Performance Indicator: Given a real number, x , a student will be able to determine all solutions, $0^\circ \leq \theta < 360^\circ$ or $0 \leq \theta < 2\pi$, for the following: $\arcsin x$, $\arccos x$, $\arctan x$, $\text{arccot } x$, $\text{arcsec } x$, and $\text{arccsc } x$ without the use of a calculator.

9.1.3 Performance Indicator: Given a simple expression involving arc trigonometric and trigonometric functions, the student will be able to evaluate the expression.

Standard 10 - Students demonstrate an understanding of the addition formulas for sines and cosines and their proofs and can use those formulas to prove and/ or simplify other trigonometric identities.

10.1 Objective: Knowledge of angle addition formulas.

10.1.1 Performance Indicator: Give two angles, θ & ϕ , whose measures are a positive or negative multiple of 30° ($\frac{\pi}{6}$) or 45° ($\frac{\pi}{4}$), the student will be able to determine the values of the following: $\sin(\theta + \phi)$, $\cos(\theta + \phi)$, $\tan(\theta + \phi)$, without the use of a calculator.

10.1.2 Performance Indicator: Given two angles, θ & ϕ , whose measures are a positive or negative multiple of 30° ($\frac{\pi}{6}$) or 45° ($\frac{\pi}{4}$), the student will be able to prove the values of the following: $\sin(2\theta) = \sin(\theta + \theta)$, $\cos(2\theta) = \cos(\theta + \theta)$, and $\tan(2\theta) = \tan(\theta + \theta)$, without the use of a calculator.

Standard 11 - Students demonstrate an understanding of half-angle and double-angle formulas for sines and cosines and can use those formulas to prove and/ or simplify other trigonometric identities.

11.1 Objective: Knowledge of half angle and double angle formulas.

11.1.1 Performance Indicator: Given two angles, θ and ϕ whose measures are a positive or negative multiple of 30° ($\frac{\pi}{6}$) or 45° ($\frac{\pi}{4}$), the student will be able to determine the values of the following: $\sin(2\theta)$, $\cos(2\theta)$, $\tan(2\theta)$, $\sin(\frac{1}{2}\theta)$, $\cos(\frac{1}{2}\theta)$, and $\tan(\frac{1}{2}\theta)$ without the use of a calculator.

11.1.2 Performance Indicator: Given two angles, θ & ϕ , whose measures are a positive or negative multiple of 30° ($\frac{\pi}{6}$) or 45° ($\frac{\pi}{4}$), the student will be able to determine the values of the following: $\sin(2\theta) = \sin(\theta + \theta)$, $\cos(2\theta) = \cos(\theta + \theta)$, and $\tan(2\theta) = \tan(\theta + \theta)$, without the use of a calculator.

Standard 12 - Students use trigonometry to determine unknown sides or angles in right triangles.

12.1 Objective: Knowledge of right triangle trigonometry.

12.1.1 Performance Indicator: Given a right triangle and the measure of two of its sides, the student will be able to write and solve an equation to find one of the unknown sides or angles.

12.1.2 Performance Indicator: Given a right triangle and the measure of one of its sides and one of its acute angles, the student will be able to write and solve an equation to find one of the unknown sides or angles.

12.1.3 Performance Indicator: Given an applied problem, the student will be able to solve the problem using right triangle trigonometry.

Standard 13 - Students know the law of sines and the law of cosines and apply those laws to solve problems.

13.1 Objective: Solve problems with oblique triangles.

13.1.1 Performance Indicator: Given the three sides of an oblique triangle, the student will be able to write and solve an equation to determine a missing angle.

13.1.2 Performance Indicator: Given two sides and the included angle of an oblique triangle, the student will be able to write and solve an equation to determine the missing side.

13.1.3 Performance Indicator: Given two angles and one side of an oblique triangle, the student will be able to write and solve equations to determine a missing side.

13.1.4 Performance Indicator: Given two sides and an angle opposite one of these two sides of an oblique triangle, the student will be able to write and solve an equation to determine the angle opposite the second given side.

Standard 14 - Students determine the area of a triangle, given one angle and the two adjacent sides.

14.1 Objective: Determining the area of a triangle.

14.1.1 Performance Indicator: Given two sides of a right triangle, the student will be able to write and solve equations to find the area of the triangle.

14.1.2 Performance Indicator: Given one side and one acute angle of a right triangle, the student will be able to write and solve equations to determine the area of the triangle.

14.1.3 Performance Indicator: Given the three sides of an oblique triangle, the student will be able to write and solve equations to determine the area of the triangle.

14.1.4 Performance Indicator: Given two angles and a side of an oblique triangle, the student will be able to write and solve equations to determine the area of the triangle.

14.1.5 Performance Indicator: Give two sides and an included angle of an oblique triangle, the student will be able to write and solve equations to determine the area of the triangle.

Standard 15 - Students are familiar with polar coordinates. In particular, they can determine polar coordinates of a point given in rectangular coordinates and vice versa.

15.1 Objective: Knowledge of polar and rectangular coordinates.

15.1.1 Performance Indicator: Given a number in rectangular form, $z = (a, b)$, the student will be able to express the number in polar form,

$$z = (r \cos \theta, r \sin \theta), \text{ where } r^2 = a^2 + b^2 \text{ and } \tan \theta = \frac{b}{a}.$$

15.1.2 Performance Indicator: Given a number in polar form, $z = (r \cos \theta, r \sin \theta)$,

where $r^2 = a^2 + b^2$ and $\tan \theta = \frac{b}{a}$, the student will be able to express the number in rectangular form.

Standard 16 - Students represent equations given in rectangular coordinates in terms of polar coordinates.

16.1 Objective: Conversion of rectangular equations to polar equations.

16.1.1 Performance Indicator: Given an equation in rectangular form, the student will be able to express the equation in polar form, by replacing 'x' by ' $r \cos \theta$ ' and 'y' by ' $r \sin \theta$ ', where $r^2 = a^2 + b^2$. (For example, the rectangular equation $y = x^2$ can be written in polar form as $r \sin \theta = (r \cos \theta)^2$).

Standard 17 - Students are familiar with complex numbers. They can represent a complex number in polar form and know how to multiply complex numbers in their polar form.

17.1 Objective: Knowledge of complex numbers in polar form.

17.1.1 Performance Indicator: Given a complex number in rectangular form, $z = a + bi$, the student will be able to express the complex number in polar form, $z = r(\cos \theta + i \sin \theta)$, where $r^2 = a^2 + b^2$.

17.1.2 Performance Indicator: Given a complex number in polar form, $z = r(\cos \theta + i \sin \theta)$, the student will be able to express the complex number in rectangular form, $z = a + bi$.

17.1.3 Performance Indicator: Given a complex number in rectangular form, $z = a + bi$, the student will be able to graph it on the complex plane.

17.1.4 Performance Indicator: Given a complex number in polar form, $z = r(\cos \theta + i \sin \theta)$, the student will be able to graph it in polar coordinates.

17.1.5 Performance Indicator: Given two complex numbers in polar form, z_1 & z_2 , the student will be able to find their product by using theorem, $z_1 z_2 = r_1 r_2 [\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)]$.

17.1.6 Performance Indicator: Given two complex numbers in polar form, z_1 & z_2 , the student will be able to find their quotient by using the theorem, $\frac{z_1}{z_2} = \frac{r_1}{r_2} [\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)]$.

Standard 18 - Students know DeMoivre's theorem and can give n^{th} roots of a complex number given in polar form.

18.1 Objective: Knowledge of DeMoivre's Theorem.

18.1.1 Performance Indicator: Given a complex number in polar form, the student will be able to find a power of the complex number by using DeMoivre's Theorem.

18.1.2 Performance Indicator: Given a complex number in polar form, the student will be able to find the n^{th} roots of the complex number by using De Moivre's Theorem.

Standard 19 - Students are adept at using trigonometry in a variety of applications and word problems.

19.1 Objective: Using trigonometry to solve application and word problems.

19.1.1 Performance Indicator: Given an application problem, the student will be able to solve the applied problem involving right triangle trigonometry.

19.1.2 Performance Indicator: Given a word problem such as nautical bearings, the student will be able to solve the problem involving right triangle trigonometry.

Linear Algebra Standards

Standard 6 – Students demonstrate an understanding that linear systems are inconsistent (have no solution), have exactly one solution, or have infinitely many solutions.

6.1 Objective: Knowledge of relations and functions

6.1.1 Performance Indicator: Students will have a working knowledge of the following vocabulary: relation, function, domain, and range.

6.1.2 Performance Indicator: Given the graph of a relation, students will be able to determine if it is a function, its domain, and its range.

6.1.3 Performance Indicator: Given the equation of a function, students will be able to express it in function notation.

6.1.4 Performance Indicator: Given a function in function notation, students will be able to evaluate the function for specific values of the independent variable.

6.1.5 Performance Indicator: Given two functions, students will be able to determine their composition.

6.1.6 Performance Indicator: Given a function, students will be able to determine its inverse.

6.2 Objective: Solve a system of linear equations and inequalities (in two or three variables) using the graphing method.

6.2.1 Performance Indicator: Given a system of equations or inequalities, such as $x - 2y = 1$ and $5x - 4y = -23$, the student will be able to find any possible solution by graphing the lines or inequalities.

6.2.2 Performance Indicator: Given a system of equations, the student will be able to determine if the system has exactly one solution, no solutions, or infinitely many solutions.

6.3 Objective: Solve a system of linear equations (in two or three variables) using the substitution method.

6.3.1 Performance Indicator: Given a system of equations such as $x + y = 4$ and $x - y = 2$, the student will be able to find any possible solution by using the substitution method for the lines or inequalities.

6.4 Objective: Solve a system of linear equations (in two or three variables) using Cramer's Rule.

6.4.1 Performance Indicator: Given a system of equations such as $x - 2y + 3z = 9$, $x - 3y = 4$, and $2x - 5y + 5z = 17$, the student will be able to find any possible solution by Cramer's Rule.

Standard 8 – Students interpret geometrically the solution sets of systems of equations. For example, the solution set of a single linear equation in two variables is interpreted as a line in the plane, and the solution set of a two-by-two system is interpreted as the intersection of a pair of lines in the plane.

8.1 Objective: Knowledge of equations, systems of equations, and functions

8.1.1 Performance Indicator: Given the equation of a simple polynomial function, students will be able to sketch its graph by plotting points.

8.1.2 Performance Indicator: Given the equation of a polynomial function whose zeros can be determined by synthetic division, factoring, or the quadratic formula, students will be able to determine its zeros.

8.1.3 Performance Indicator: Given the graph of a polynomial function, students will be able to identify its zeros, relative maximum points, relative minimum points, domain, and range.

8.1.4 Performance Indicator: Given the graph of a simple polynomial function, students will be able to write one equation which satisfies the conditions of the graph.

8.1.5 Performance Indicator: Given the equation of a rational function, students will be able to determine its x - and y -intercepts.

8.1.6 Performance Indicator: Given the equation of a rational function, students will be able to determine the equations of its vertical, horizontal, and oblique asymptotes, and sketch its graph.

8.1.7 Performance Indicator: Given a system of linear equations, students will be able to solve by graphing each equation and know the intersection is the solution.

Mathematical Analysis Standards

Standard 4 – Students know the statement of, and can apply the fundamental theorem of algebra.

4.1 Objective: Finding quadratic or cubic functions given the zeros.

4.1.1 Performance Indicator: Given the zeros of a function, the student will be able to write the equation.

Standard 5 – Students are familiar with conic sections, both analytically and geometrically.

5.1 Objective: Students can take a quadratic equation in two variables; put it in standard form by completing the square and determine what type of conic section the equation represents, and determine its geometric components (foci, asymptotes, vertices, center)

5.1.1 Performance Indicator: Given the equation of a conic in general or standard form, students will be able to determine which conic it is.

5.1.2 Performance Indicator: Given the equation of the form $Ax^2 + By^2 + Cx + Dy + E = 0$, students will be able to write the equation in standard form.

5.1.3 Performance Indicator: Given the equation of the form $Ax^2 + By^2 + Cx + Dy + E = 0$, students will be able to graph the conic.

Probability and Statistics Standards

Standard 1 – Students know the definition of the notion of *independent events* and can use the rules for addition, multiplication, and complementation to solve for probabilities of particular events in finite sample spaces.

1.0 Objective: Knowledge of Independent events and probability computations

1.1 Performance Indicator: The student will be able to define the probability of an event occurring is the relative frequency with which that event is likely to occur.

1.2 Performance Indicator: Given the number of outcomes corresponding to the chosen event and the total number of possible outcomes, the student will be able to calculate the probability of an event occurring.

- 1.3 Performance Indicator: Given the appropriate information, the student will be able to recognize mutually exclusive events and calculate their probability using the addition rule.
- 1.4 Performance Indicator: Students will know that two events, A and B, are independent if the outcome of one event, A, does not influence or change the outcome of the other event, B.
- 1.5 Performance Indicator: Given the appropriate information, the student will be able to recognize independent events and calculate their probabilities using the multiplication rule.
- 1.6 Performance Indicator: Students will know for two independent events, A and B, the probability that both A *and* B occur is the product of the probabilities of the two events. $P(A \text{ and } B) = P(A \cap B) = P(A) \cdot P(B)$
- 1.7 Performance Indicator: Students will know that the complement of an event, A, occurring is one minus the probability that the event does not occur, $P(A) = 1 - P(A^c)$.

Standard 2 - Students know the definition of the notion of *conditional probability* and use it to solve for probabilities in finite sample spaces.

2.0 Objective: Knowledge of conditional probability

- 2.1 Performance Indicator: Students will be able to recognize the probability from an event with a conditional distribution.
- 2.2 Performance Indicator: Students will know that a probability that takes into account a given condition is called a conditional probability.
- 2.3 Performance Indicator: Students will know that a conditional probability of the event B *given* event A has occurred is calculated by restricting the outcomes to event A and finding the fraction of those outcomes where event B also occurs. $P(B|A) = \frac{P(B \cap A)}{P(A)}$

Standard 7 - Students compute the variance and the standard deviation of a distribution of data.

7.0 Objective: Knowledge of data analysis

- 7.1 Performance Indicator: Given a set of data, students will be able to calculate the measures of central tendency of the data, such as mean, median, and mode.

7.2 Performance Indicator: Given a set of data, students will be able to calculate the measures of spread of the data, such as range, variation, and standard deviation.