Chapter 6

6.2 The Law of Cosines

Chapter 6.2

Homework

- Read Sec 6.2
- Do p443 1-43 odd

Chapter 6.2

Objectives

- Use the Law of Cosines to solve oblique triangles.
- Solve applied problems using the Law of Cosines.
- Use Heron's formula to find the area of a triangle.

Reminder

Draw a Picture

Solving Oblique Triangles

Objective: Students use the Law of Cosines to solve triangles.

- Solving an oblique triangle means finding the lengths of its sides and the measures of its angles.
  - We used the Law of Sines to solve SSA, ASA, and AAS triangles.

Law of Cosines

- The Law of Cosines is used to solve triangles when we know ...
  - two sides and the included angle (SAS), or three sides (SSS).

The Law of Cosines

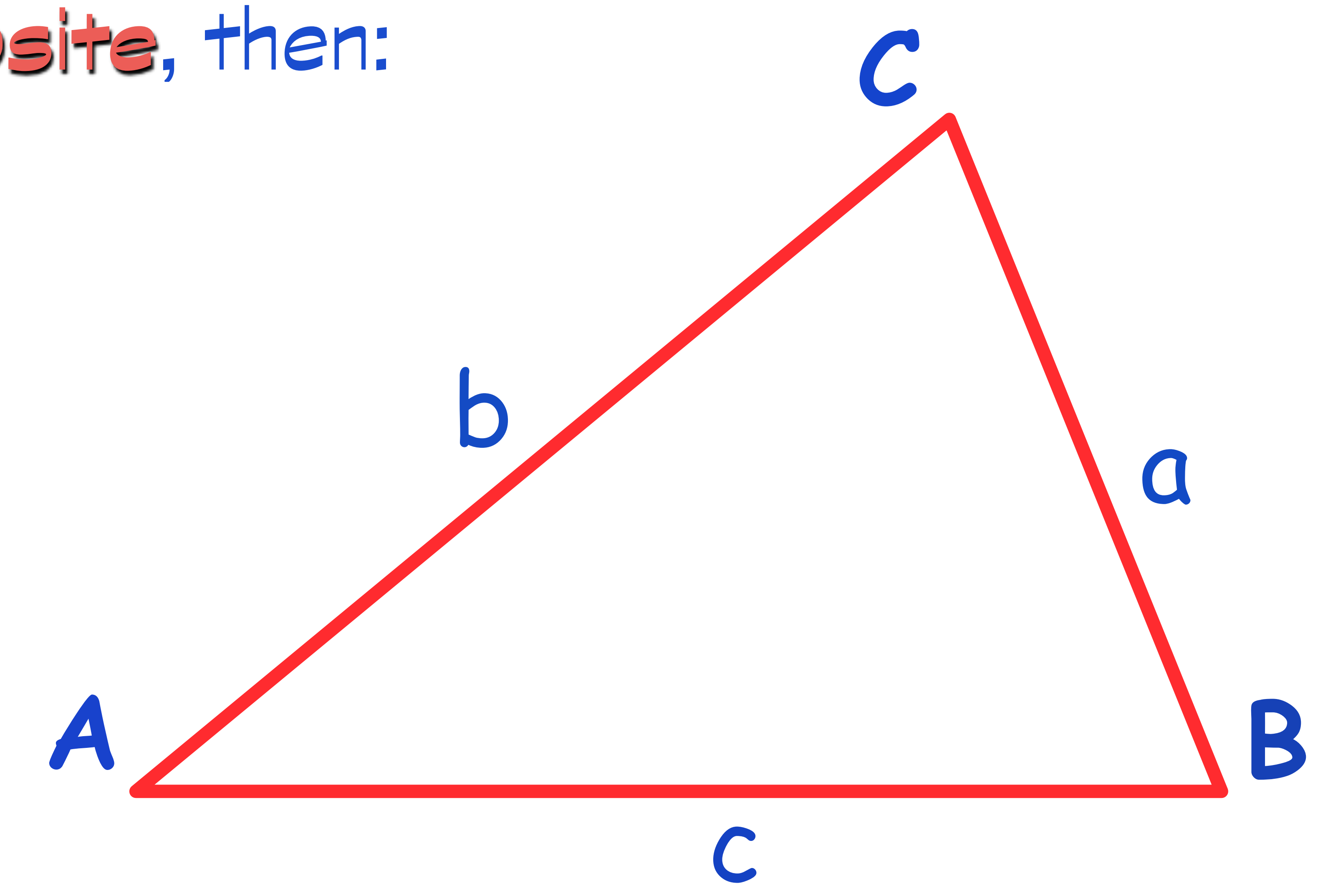
Objective: Students use the Law of Cosines to solve triangles.

- Let us take our typical oblique triangle ABC.
  - If A, B, and C are the measures of the angles of a triangle, and a, b, and c are the lengths of the sides opposite, then:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

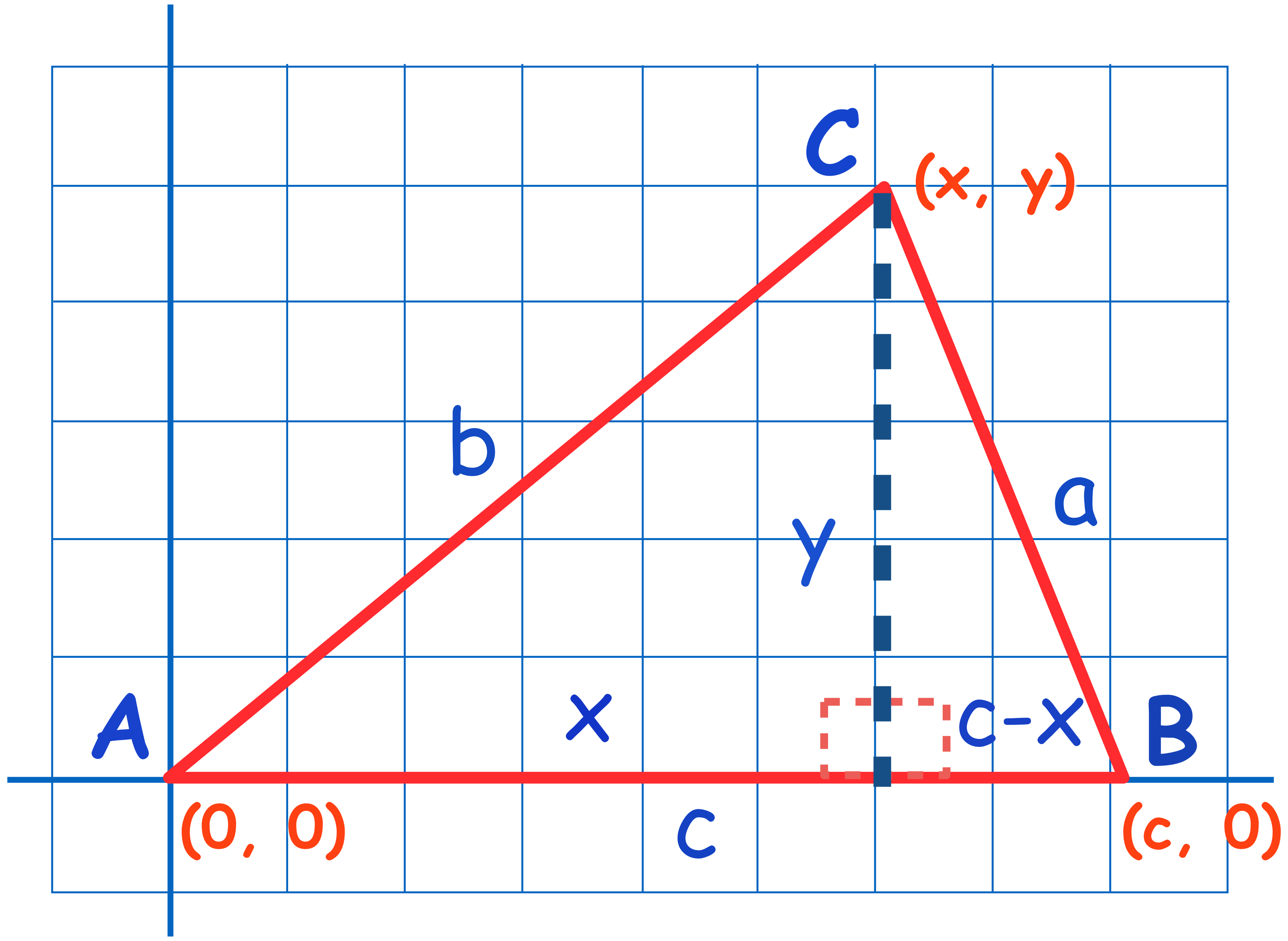


- Look vaguely familiar?

Law of Cosines

Objective: Students use the Law of Cosines to solve triangles.

- Now lay the coordinate plane over ΔABC so vertex A is at (0, 0) and drop a height to one of the sides. I chose Vertex C to side c but we could use any vertex



$\cos A = \frac{x}{b}, \ x = b\cos A \qquad \sin A = \frac{y}{b}, \ y = b\sin A$

$a^2 = y^2 + (c-x)^2$ Pythagorean Theorem

$a^2 = b^2\sin^2 A + (c - b\cos A)^2$ Substitution

$a^2 = b^2\sin^2 A + c^2 - 2cb\cos A + b^2\cos^2 A$

$a^2 = b^2\sin^2 A + b^2\cos^2 A + c^2 - 2cb\cos A$

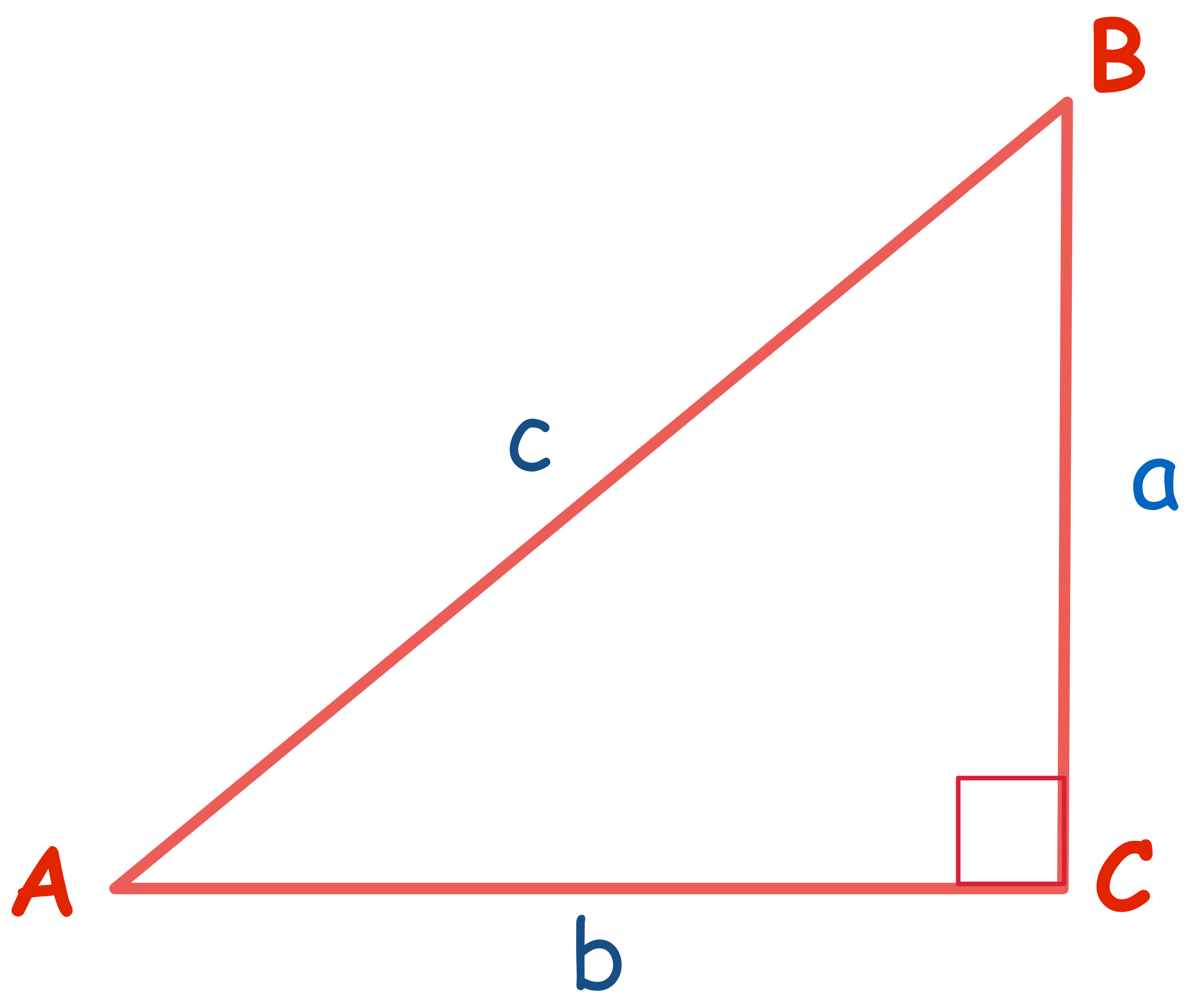
$a^2 = b^2(\sin^2 A + \cos^2 A) + c^2 - 2cb\cos A$

$a^2 = b^2(1) + c^2 - 2cb\cos A$

Pythagorus

Objective: Students use the Law of Cosines to solve triangles.

- Consider the case of the right triangle and the Pythagorean Theorem.



$$c^2 = a^2 + b^2 - 2ab\cos C$$

Since c = 90°, cos C = 0.

$$c^2 = a^2 + b^2$$

- The Pythagorean Theorem is a special case of the Law of Cosines.

Law of Cosines

Objective: Students use the Law of Cosines to solve triangles.

- The Law of Cosines can also be written from the perspective of the angle.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

- A little algebra can rewrite the Law of Cosines as:

$$2bc\cos A = b^2 + c^2 - a^2$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Law of Cosines

Objective: Students use the Law of Cosines to solve triangles.

- So to complete the set

$$a^2 = b^2 + c^2 - 2bc\cos A \qquad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$b^2 = a^2 + c^2 - 2ac\cos B \qquad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$c^2 = a^2 + b^2 - 2ab\cos C \qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Solving an SAS Triangle

Objective: Students use the Law of Cosines to solve triangles.

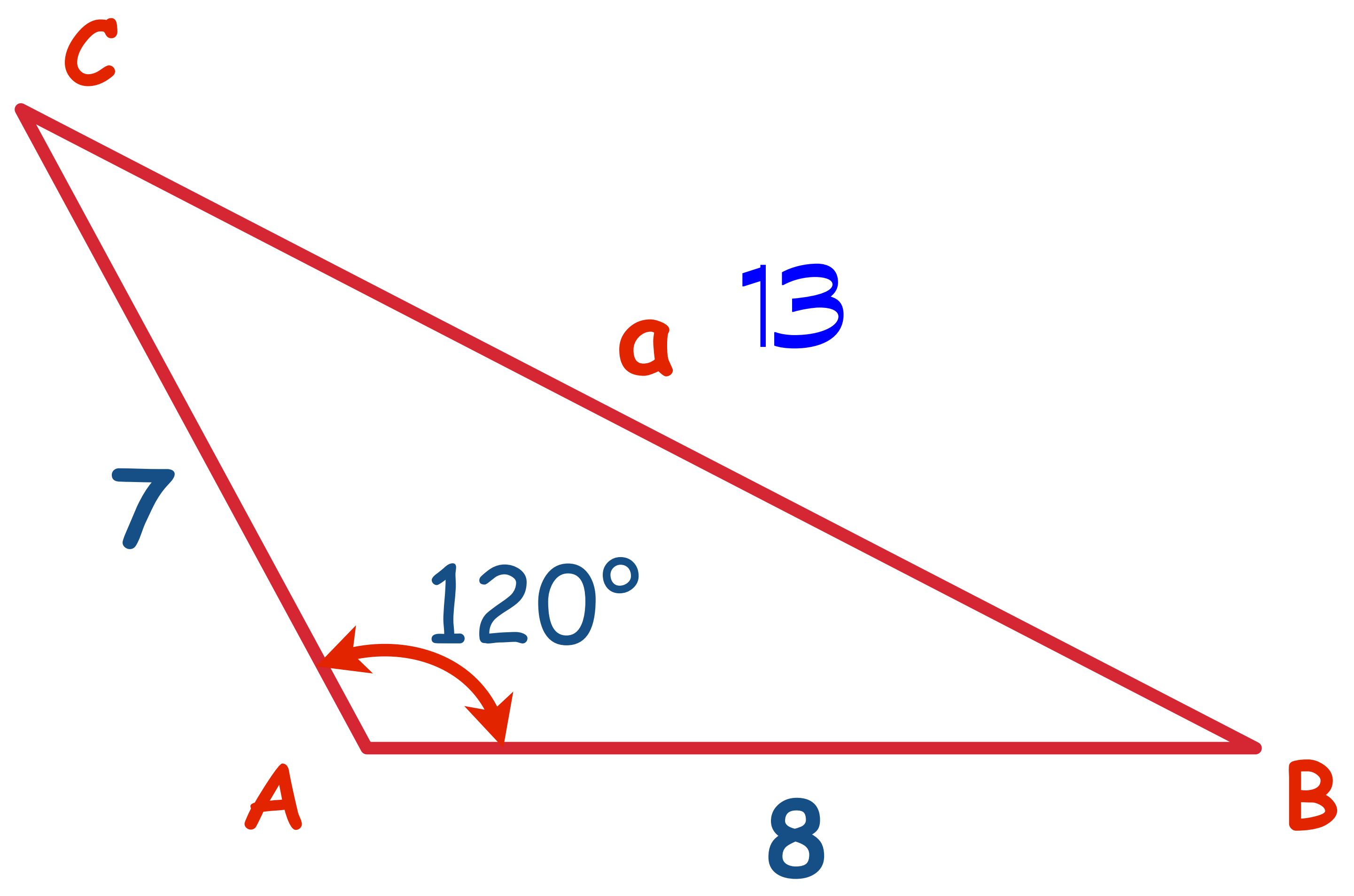
Solving an oblique triangle when we have two sides and the included angle (SAS).

1. Use Law of Cosines to find the side opposite the given angle.
2. Use Law of Sines to find angle opposite the shorter given side.
3. Find the third angle (sum = 180°).

Solving an SAS Triangle

Objective: Students use the Law of Cosines to solve triangles.

- Solve the triangle shown in the figure with A = 120°, b = 7, and c = 8 (SAS). Round lengths of sides to the nearest tenth and angle measures to the nearest degree.



1. Use the Law of Cosines.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a^2 = 7^2 + 8^2 - 2 \bullet 7 \bullet 8\cos 120°$$

$$a^2 = 7^2 + 8^2 - 2 \bullet 7 \bullet 8 \bullet \left(-\frac{1}{2}\right) = 169$$

$$a = 13$$

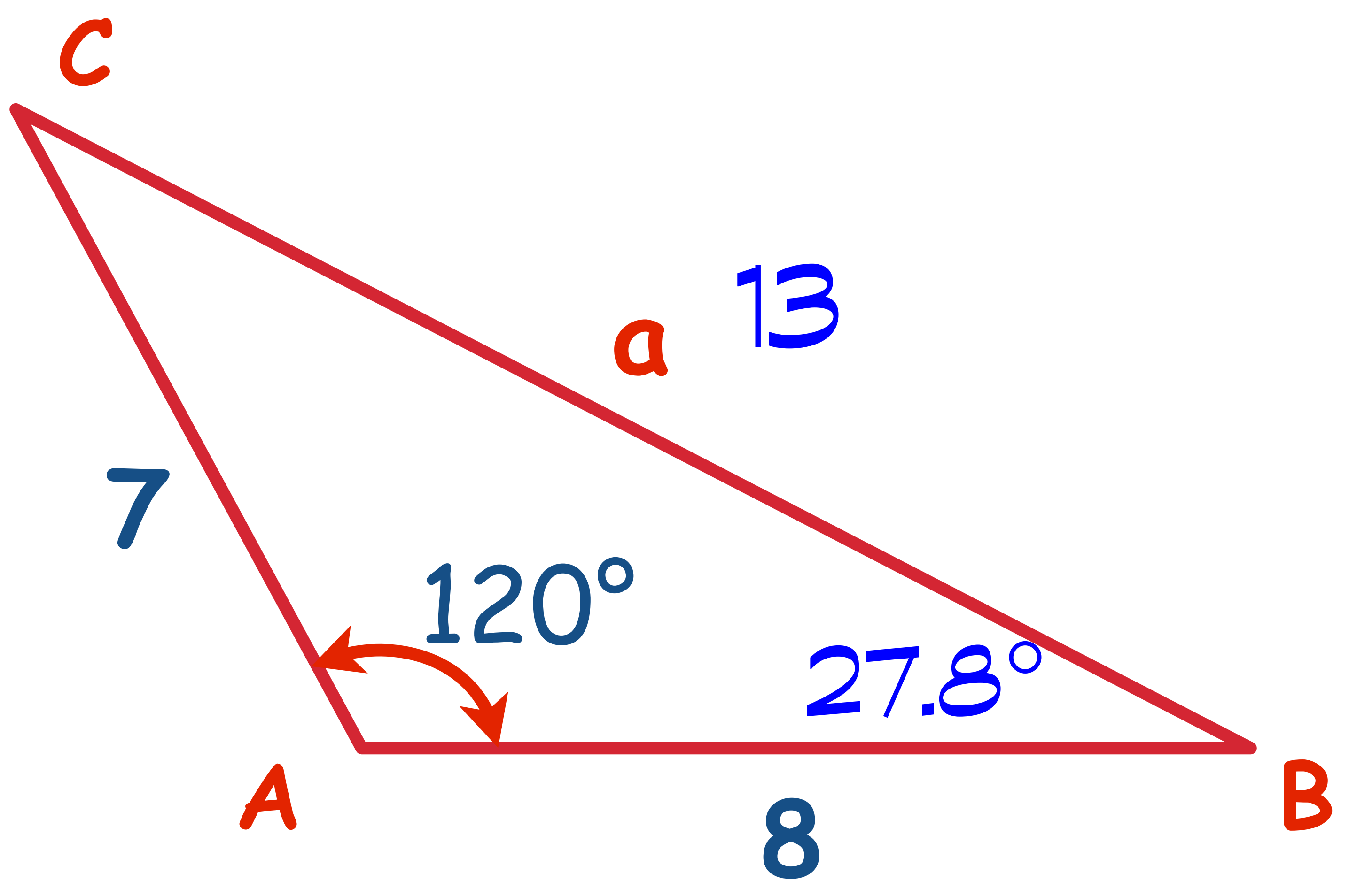
Note the appropriate relationships between the angles of the triangle and sides opposite!

Solving an SAS Triangle

Objective: Students use the Law of Cosines to solve triangles.

- Solve the triangle shown in the figure with $A = 120°$, $b = 7$, and $c = 8$. Round lengths of sides to the nearest tenth and angle measures to the nearest degree.

$$a = 13$$



2. Use the Law of Sines.

$$\frac{7}{\sin B} = \frac{13}{\sin 120°} \qquad \sin B = \frac{7 \sin 120°}{13}$$

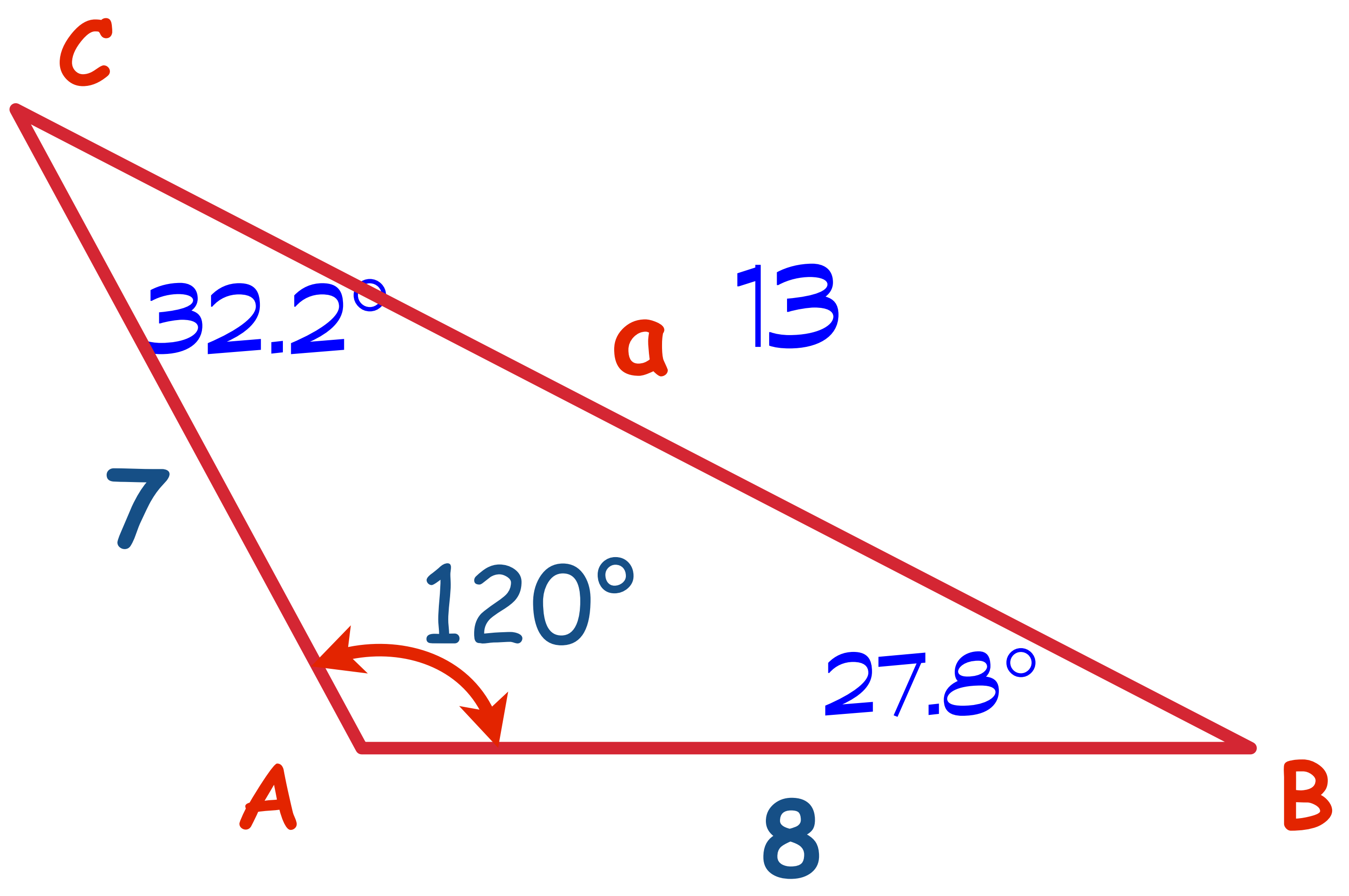
$$B = \sin^{-1}\left(\frac{7 \sin 120°}{13}\right) \qquad B \approx 27.8°$$

Solving an SAS Triangle

Objective: Students use the Law of Cosines to solve triangles.

- Solve the triangle shown in the figure with $A = 120°$, $b = 7$, and $c = 8$. Round lengths of sides to the nearest tenth and angle measures to the nearest degree.

$$a = 13 \quad B \approx 27.8°$$



3. Find the third angle.

$$C \approx 180° - 120° - 27.8° \approx 32.2°$$

$A = 120°$, $B \approx 28°$, $C \approx 32°$, $a = 13$, $b = 7$, $c = 8$

Note the appropriate relationships between the angles of the triangle and sides opposite!

Solving an SSS Triangle:

Objective: Students use the Law of Cosines to solve triangles.

Solving an oblique triangle when we are given three sides (SSS).

1. Use Law of Cosines to find the angle **opposite the longest side.**

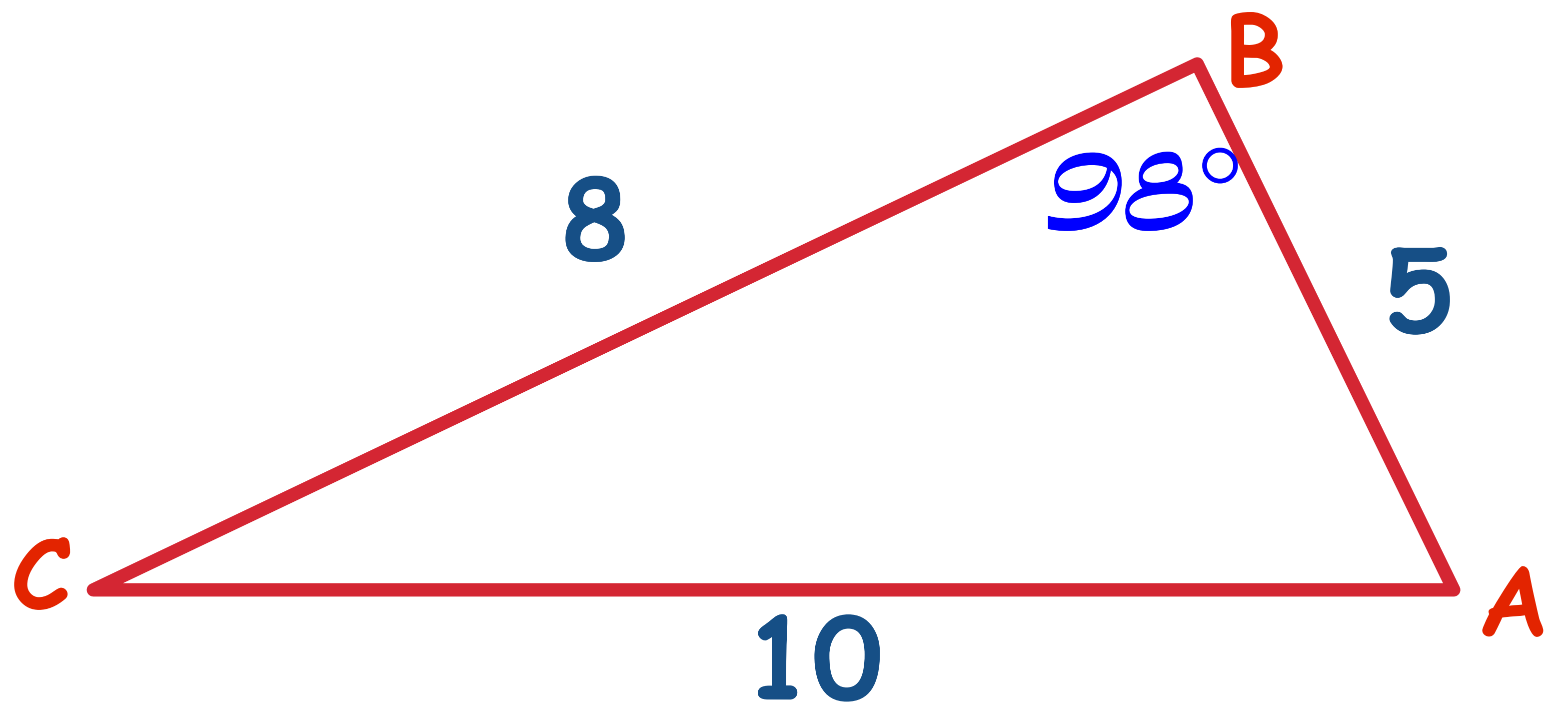
2. Use Law of Sines to find another (any) angle(s).

3. Find the third angle (sum = 180°).

Solving an SSS Triangle:

Objective: Students use the Law of Cosines to solve triangles.

- Solve ΔABC if $a = 8$, $b = 10$, and $c = 5$. Round angle measures to the nearest degree.



Note: $8^2 + 5^2 < 10^2$

Thus $B > 90°$

1. Use Law of Cosines to find the angle opposite the longest side.

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$10^2 = 8^2 + 5^2 - 2(5)(8)\cos B$$

$$100 = 64 + 25 - 80\cos B$$

$$-80\cos B = 11 \qquad \cos B = \frac{11}{-80}$$

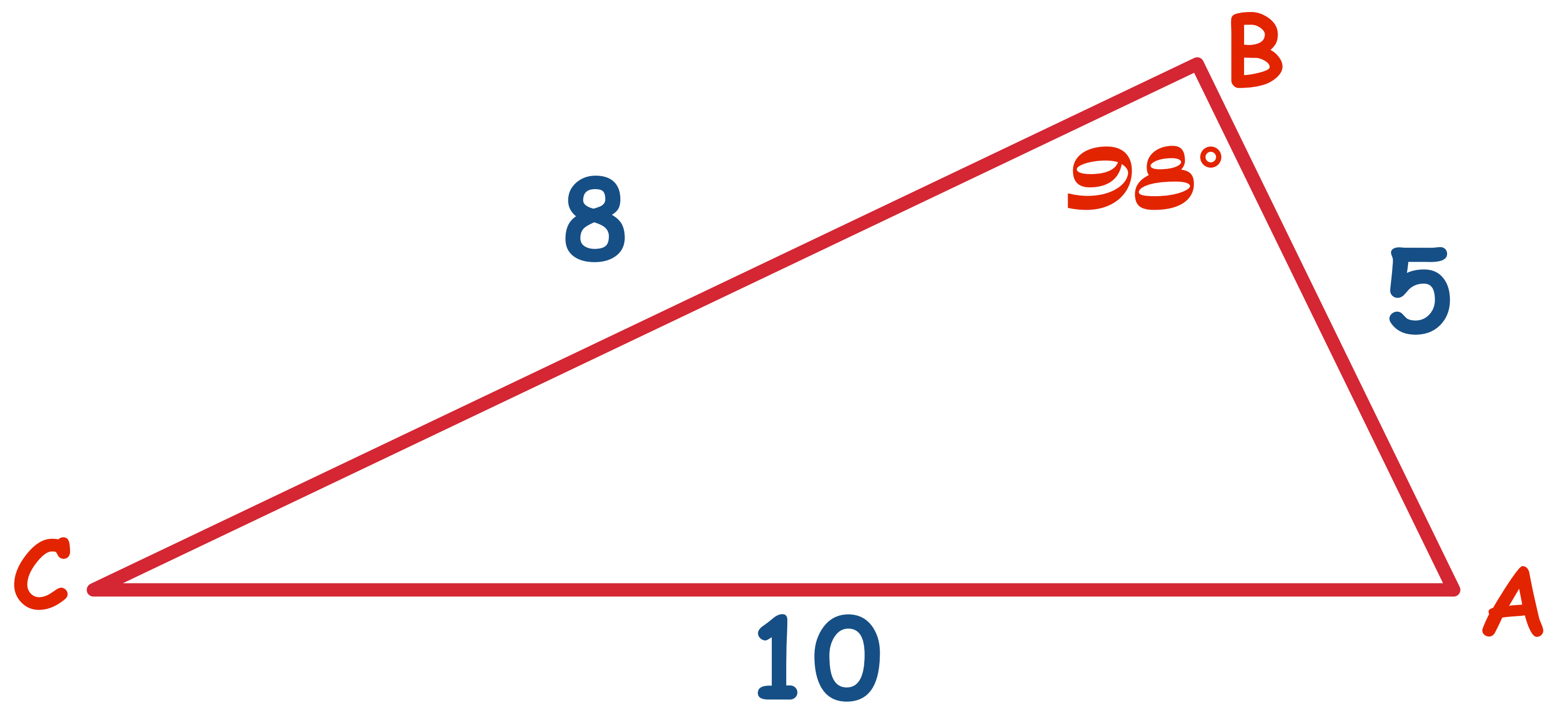
$$B = \cos^{-1}\left(-\frac{11}{80}\right) \approx 97.9° \qquad B \approx 98°$$

Solving an SSS Triangle:

Objective: Students use the Law of Cosines to solve triangles.

- Solve ΔABC if $a = 8$, $b = 10$, and $c = 5$. Round angle measures to the nearest degree.

$B \approx 98^\circ$



2. Use Law of Sines to find another (any) angle(s).

$$\frac{5}{\sin C} = \frac{10}{\sin 98^\circ} \qquad \sin C = \frac{5 \sin 98^\circ}{10}$$

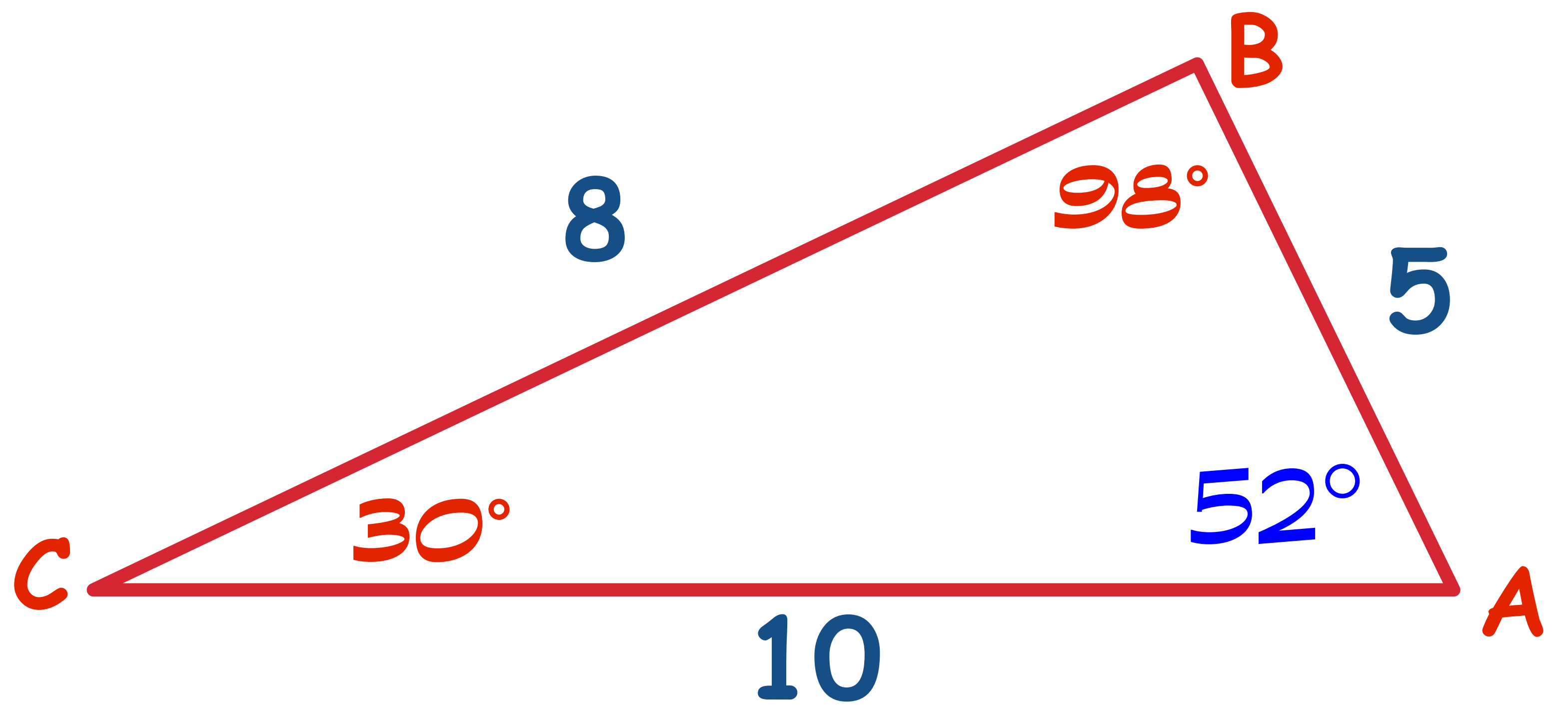
$$C = \sin^{-1}\left(\frac{5 \sin 98^\circ}{10}\right) \qquad C \approx 29.68^\circ \approx 30^\circ$$

Solving an SSS Triangle:

Objective: Students use the Law of Cosines to solve triangles.

Solve $\triangle ABC$ if $a = 8$, $b = 10$, and $c = 5$. Round angle measures to the nearest degree.

$B \approx 98^\circ \quad C \approx 29.68^\circ \approx 30^\circ$



3. Find the third angle.

$A = 180^\circ - 98^\circ - 30^\circ = 52^\circ$

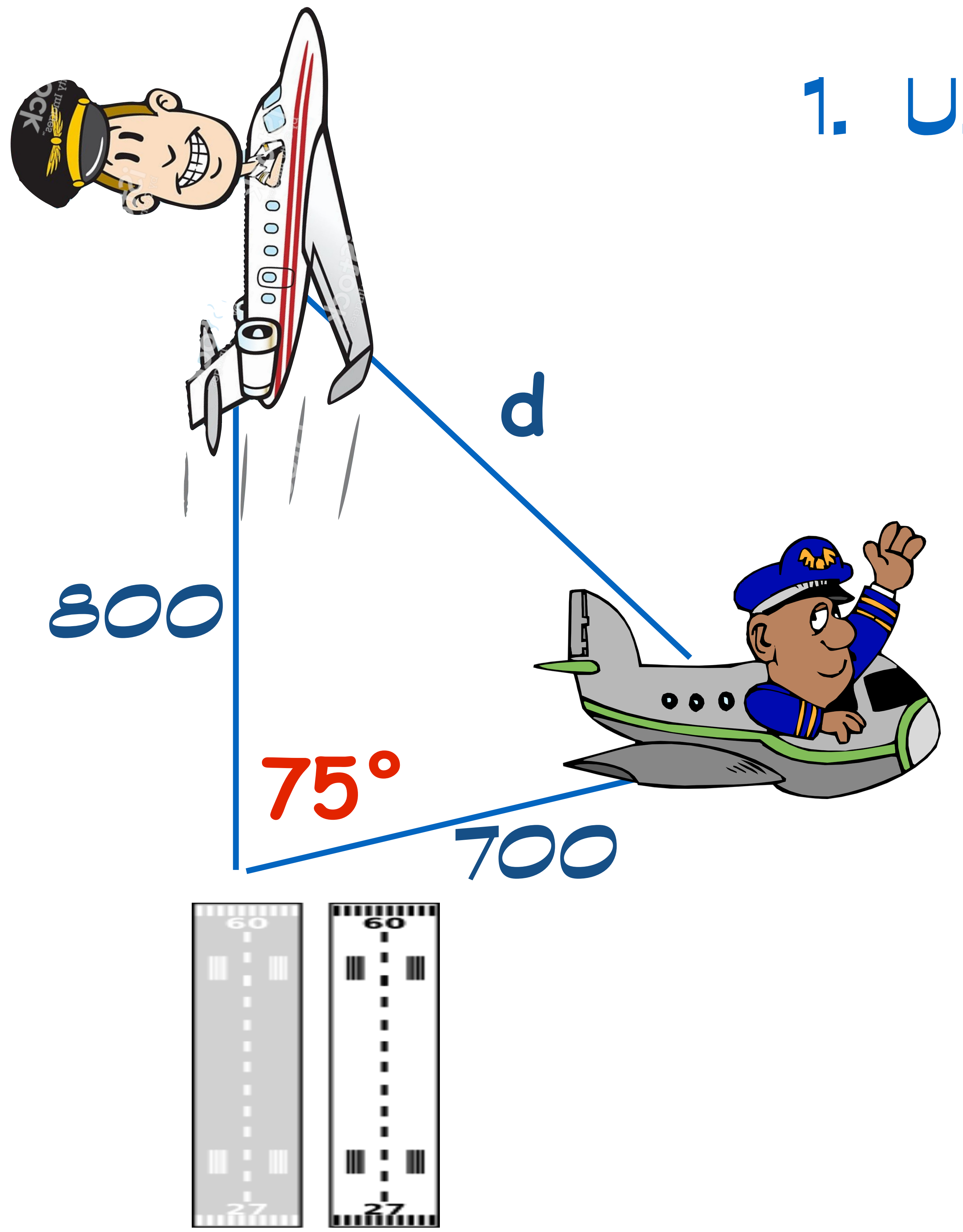
$A \approx 52^\circ$, $B \approx 98^\circ$, $C \approx 30^\circ$, $a = 8$, $b = 10$, $c = 5$

Note the appropriate relationships between the angles of the triangle and sides opposite!

Application

Objective: Students use the Law of Cosines to solve triangles.

- Two airplanes leave an airport at the same time on different runways. One flies directly north at 400 miles per hour. The other airplane flies on a bearing of N75°E at 350 miles per hour. How far apart will the airplanes be after two hours?



1. Use Law of Cosines to find the side opposite the angle.

$$d^2 = 800^2 + 700^2 - 2(800)(700)\cos 75^\circ$$

$$d^2 = 640000 + 490000 - 1120000)(0.258819)$$

$$d^2 \approx 840122.72 \qquad d \approx 916.58$$

The planes will be approximately 916.6 miles apart.

Heron's Formula

Objective: Students use the Law of Cosines to solve triangles.

Heron's Formula for the area of a triangle.

The area of a triangle with sides a, b, c is:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

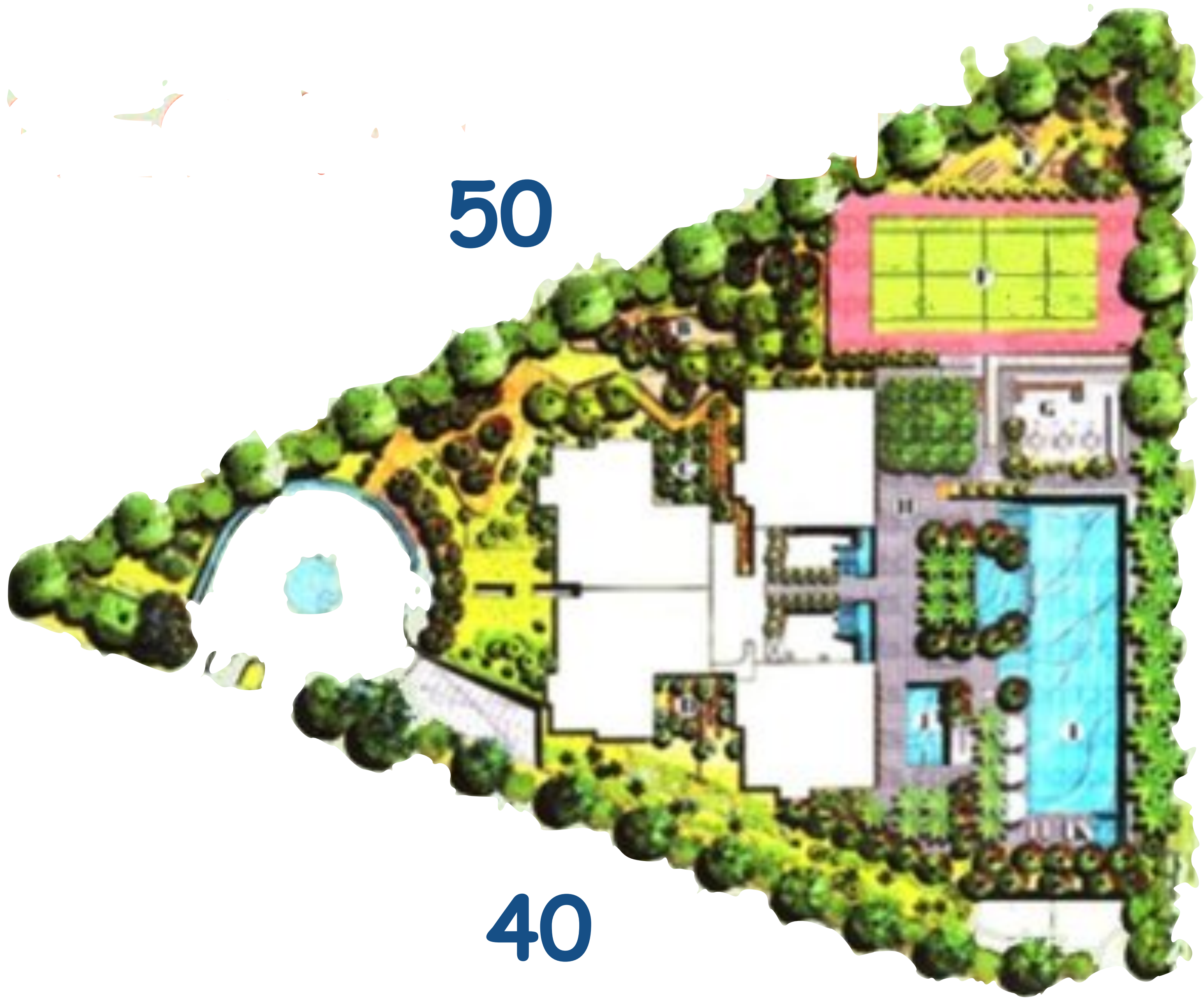
Where s = one-half the perimeter of the triangle.

$$s = \frac{1}{2}(a+b+c)$$

Heron's Formula

Objective: Students use the Law of Cosines to solve triangles.

- You are looking at possibly purchasing a triangular shaped lot but you think the owner has exaggerated the area of the lot. The lot is 40 yards by 50 yards by 30 yards. Find the area.



Perimeter = 120 yards

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{1}{2}(120) = 60$$

$$A = \sqrt{60(60-50)(60-40)(60-30)}$$

$$A = \sqrt{360000} = 600\ yd^2$$