

Compound Inequalities

AND/OR Problems

There are two types of compound inequalities. They are conjunction problems and disjunction problems. These compound inequalities will sometimes appear as two simple inequalities separated by using the word **AND** or **OR**. When solving “and/or” compound inequalities, begin by solving each inequality individually. The solution will come when we determine whether any area shaded is the solution area, or is the overlap the solution area.

Follow these rules for “and/or” problems:

Conjunction problems use the word and while disjunction problems use the word or.

For conjunction problems, we are looking for an overlap of the graphs to determine the solution interval.

For disjunction problems, any area shaded is part of the solution interval.

The next few pages contain examples of how to find the solution to these types of compound inequalities.

Example 1

$$2x - 5 \geq 3 \text{ and } x + 4 \leq 7$$

Begin by separating the two inequalities and solving each individually.

$$2x - 5 \geq 3 \quad \text{and} \quad x + 4 \leq 11$$

$$2x - 5 \geq 3 \quad \text{and} \quad x + 4 \leq 11$$

$$+5 \quad +5 \quad \text{and} \quad -4 \quad -4$$

$$\frac{2x}{2} \geq \frac{8}{2} \quad \quad \quad x \leq 7$$

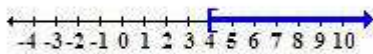
$$x \geq 4$$

In order to tell how the solution should look, let's graph each of these inequalities individually on a number line.

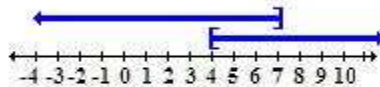
$$x \geq 4$$

and

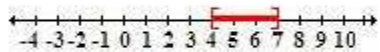
$$x \leq 7$$



In the picture below, both of the inequalities are shown above the number line. Notice how the two regions overlap between 4 and 7.



Since this is a conjunction problem using the word and, we are looking for the overlap. The solution is the area where the lines overlap. Below is the solution to this particular inequality in both graphical form and using interval notation.



$$[4, 7]$$

Since all of these problems involve solving a simple inequality, next few examples will show how to determine the solution to these types of problems. Therefore, we will skip solving the simple inequalities as by now that shouldn't be a problem.

Notice that in this example, the problem switched from an **and** problem to an **or** problem.

Example 2

$$2x - 5 \geq 3 \text{ or } x + 4 \leq 7$$

Begin by separating the two inequalities and solving each individually.

$$2x - 5 \geq 3 \quad \text{or} \quad x + 4 \leq 11$$

$$2x - 5 \geq 3 \quad \text{or} \quad x + 4 \leq 11$$

$$+5 \quad +5 \quad \text{or} \quad -4 \quad -4$$

$$\frac{2x}{2} \geq \frac{8}{2} \quad \text{or} \quad x \leq 7$$

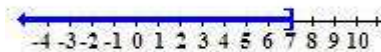
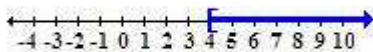
$$x \geq 4$$

In order to tell how the solution should look, let's graph each of these inequalities individually on a number line.

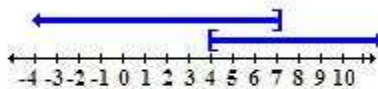
$$x \geq 4$$

or

$$x \leq 7$$



In the picture below, both of the inequalities are shown above the number line. Notice how the two regions overlap between 4 and 7.



Since this is a disjunction problem using the word **or**, we are looking for any area shaded. The solution to this inequality will be any area that has been shaded by the two individual inequalities. Since they both continue on, the solution is all real numbers. Below is the solution to this particular inequality in both graphical form and using interval notation.



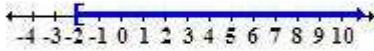
$$(-\infty, \infty)$$

Example 3

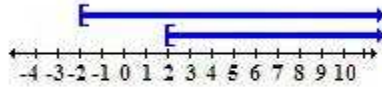
$$x \geq -2$$

and

$$x \geq 2$$



In the picture below, both of the inequalities are shown above the number line. Notice how the two regions will only overlap from 2 to infinity.



These two overlap from 2 to infinity, so the overlap is the only solution area we will use.



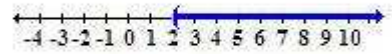
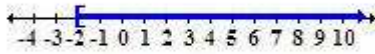
$$[2, \infty)$$

Example 4

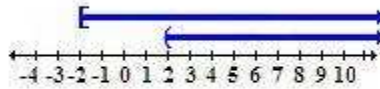
$$x \geq -2$$

or

$$x > 2$$



In the picture below, both of the inequalities are shown above the number line. Since this is an or problem, we do not need an overlap for the solution area.



We do not need an overlap in this example. As a result, the solution area will begin at -2 instead of where they begin to overlap.



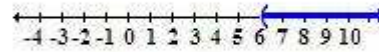
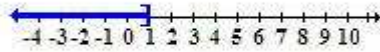
$$[-2, \infty)$$

Example 5

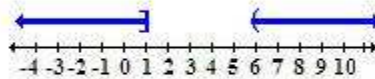
$x \leq 1$

and

$x > 6$



In the picture below, both of the inequalities are shown above the number line. There is no overlap anywhere on this graph. The solution of equalities like these depends solely on the word that is used for the compound inequality. In this case, the word and is used which means we need an overlap.



For this example, we need an overlap, but do not have one. This means, there is no solution to this problem.

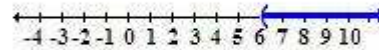
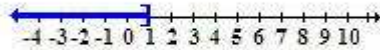
Example 6

We will use the same problem here, however, this will be an or problem.

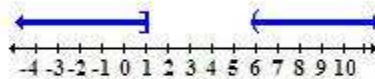
$x \leq 1$

or

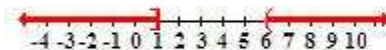
$x > 6$



In the picture below, both of the inequalities are shown above the number line. There is no overlap anywhere on the graph, therefore, the solution will depend solely on the word that is used for the compound inequality. In this case, the word or is used which means we do not need an overlap.



No overlap is needed for the or problem, so any area shaded is the solution area. Our solution area is from $-\infty$ to 1 inclusive of one, and from 6 to ∞ exclusive of six.



$$(-\infty, 1] \cup (6, \infty)$$