

# LOGARITHMS

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## Objectives

*The following is a list of objectives for this section of the workbook.*

*By the time the student is finished with this section of the workbook, he/she should be able to...*

- *Evaluate a simple logarithm without the aid of a calculator.*
- *Express a logarithmic statement in exponential form.*
- *Express a statement in exponential form in logarithmic form.*
- *Expand a logarithmic expression as the sum or difference of logarithms using the properties of logs.*
- *Condense the sum or difference of logarithms into a single logarithmic expression.*
- *Evaluate logarithms using the base change formula.*
- *Solve logarithmic equations.*
- *Evaluate the solution to logarithmic equations to find extraneous roots.*
- *Solve equations with variables in the exponents.*
- *Find the range and domain of logarithmic functions.*
- *Graph a logarithmic function using a table.*
- *Find the inverse of a function.*
- *Verify two functions are inverses of each other.*
- *Identify a one-to-one function.*
- *Use the compound interest formulas.*

## Math Standards Addressed

*The following state standards are addressed in this section of the workbook.*

### Algebra II

**11.0** Students prove simple laws of logarithms.

11.1 Students understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

11.2 Students judge the validity of an argument according to whether the properties of real numbers, exponents, and logarithms have been applied correctly at each step.

**13.0** Students use the definition of logarithms to translate between logarithms in any base.

**14.0** Students understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values.

**15.0** Students determine whether a specific algebraic statement involving rational expressions, radical expressions, or logarithmic or exponential functions is sometimes true, always true, or never true.

**24.0** Students solve problems involving functional concepts, such as composition, defining the inverse function and performing arithmetic operations on functions.

*Logarithms are similar to radicals in that knowing what the question is asking makes the problem easier. Although this is a topic that is completely new to Algebra II students, Logarithms are simple. For example, the question  $\log_3 27 =$  is asking “To what power do you raise 3 to get 27?” In this particular problem, 3 is the base of the logarithm. When reading the logarithm, it is read “Log base 3 of 27 is...”*

**Properties of Simple Logarithms**

$$\log_a 1 = 0$$

$$\log_a a = 1$$

$$\log_a a^x = x \text{ and } a^{\log_a x} = x \text{ (inverse property)}$$

$$\text{If } \log_a x = \log_a y \text{ then } x = y$$

**Properties of Natural Logarithms**

$$\ln 1 = 0$$

$$\ln e = 1$$

$$\ln e^x = x \text{ and } e^{\ln x} = x \text{ (inverse property)}$$

$$\text{If } \ln x = \ln y \text{ then } x = y$$

*A standard logarithm can have any positive number as its base except 1, whereas a natural log is always base  $e$ . Since the natural log is always base  $e$ , it will be necessary to use a calculator to evaluate natural logs unless one of the first three examples of the properties of natural logs is used. For anything such as  $\ln 2 =$ , a calculator must be used.*

*When dealing with logarithms, switching between exponential and Logarithmic form is often necessary.*

**Logarithmic form**

$$\log_a b = c$$

**Exponential Form**

$$a^c = b$$

**Write each of the following in exponential form.**

- A)  $\log_4 16 = 2$       B)  $\log_9 3 = \frac{1}{2}$       C)  $\log_9 27 = \frac{3}{2}$       E)  $\log_4 \frac{1}{16} = -2$

**Write each of the following in logarithmic form.**

- A)  $3^4 = 81$       B)  $16^{1/4} = 2$       C)  $36^{-1/2} = \frac{1}{6}$       D)  $16^{5/4} = 32$

## Simplifying Logarithms

Evaluate each of the following logarithms without the use of a calculator.

A)  $\log_3 81 =$

B)  $\log_4 \frac{1}{2} =$

C)  $\log_{12} 144 =$

D)  $\log_6 \frac{1}{36} =$

E)  $\log_{\frac{2}{3}} \frac{9}{4} =$

F)  $\log_{0.25} 4 =$

G)  $\log_3 -3 =$

H)  $\log_8 4 =$

I)  $\log_{81} \frac{1}{27} =$

J)  $\log_{\frac{1}{16}} 32 =$

K)  $\log_4 0 =$

L)  $\log_{10} 1 =$

M)  $\log_4 \frac{1}{8} =$

N)  $\log_{27} \frac{1}{3} =$

O)  $\log_9 3 =$

P)  $\log_6 6^{3x} =$

Q)  $\log_{36} \frac{1}{6} =$

R)  $\log_{128} 2 =$

S)  $\log_{\frac{1}{4}} 16 =$

T)  $\log_z z^{2x} =$

U)  $\ln e^{12} =$

V)  $3^{\log_3 5} =$

W)  $\ln 1 =$

X)  $e^{\ln 4x} =$

Y)  $\log_2 16\sqrt{2} =$

Z)  $\log_3 \sqrt[5]{9} =$

a)  $\log_3 9\sqrt[3]{3} =$

b)  $\log_5 \frac{1}{\sqrt[3]{25}} =$

c)  $\log_{\frac{5}{6}} \sqrt[3]{\frac{36}{25}} =$

d)  $e^{\ln 5x^2} =$

## Properties of Logarithms

*The following properties serve to expand or condense a logarithm or logarithmic expression so it can be worked with.*

### Properties of logarithms

$$\log_a mn = \log_a m + \log_a n$$

$$\log_a \frac{m}{n} = \log_a m - \log_a n$$

$$\log_a m^n = n \log_a m$$

### Example

$$\log_4 3x = \log_4 3 + \log_4 x$$

$$\log_2 \frac{x+1}{5} = \log_2 (x+1) - \log_2 5$$

$$\log_3 (2x+1)^3 = 3 \log_3 (2x+1)$$

### Properties of Natural Logarithms

$$\ln mn = \ln m + \ln n$$

$$\ln \frac{m}{n} = \ln m - \ln n$$

$$\ln m^n = n \ln m$$

### Example

$$\ln (x+1)(x-5) = \ln (x+1) + \ln (x-5)$$

$$\ln \frac{x}{2} = \ln x - \ln 2$$

$$\ln 7^3 = 3 \ln 7$$

*These properties are used backwards and forwards in order to expand or condense a logarithmic expression. Therefore, these skills are needed in order to solve any equation involving logarithms. Logarithms will also be dealt with in Calculus. If a student has a firm grasp on these three simple properties, it will help greatly in Calculus.*

## Expanding Logarithmic Expressions

Write each of the following as the sum or difference of logarithms. In other words, expand each logarithmic expression.

A)  $\log_2 \frac{3x^3y^2}{z^5}$

B)  $\log_3 5\sqrt[3]{xy^2}$

C)  $\log \sqrt[4]{(x+1)^3(x-2)^2}$

D)  $\log_5 \frac{6x^2}{11y^5z}$

$$\mathbf{E)} \log_2 \frac{\sqrt[5]{3(x+2)^3}}{x-1}$$

$$\mathbf{F)} \log_{12} \frac{x-7}{x+2}$$

$$\mathbf{G)} \log_a 12x^3 \sqrt{y}$$

$$\mathbf{H)} \log_3 \frac{\sqrt{5x^5 y^3}}{\sqrt[3]{z^2}}$$

$$\mathbf{I)} \ln \frac{x^2 - 4}{x^3}$$

$$\mathbf{J)} \ln 3x^5 y$$

$$\mathbf{K)} \ln \sqrt{x^3(x+4)}$$

$$\mathbf{L)} \ln \sqrt[3]{x^2(x-3)}$$

$$\mathbf{M)} \ln \sqrt{\frac{x^3 y}{z^5}}$$

$$\mathbf{N)} \ln \frac{x}{\sqrt{x-2}}$$

### Condensing Logarithmic Expressions

Rewrite each of the following logarithmic expressions using a single logarithm. Condense each of the following to a single expression. Do not multiply out complex polynomials. Just leave something like  $(x+5)^3$  alone.

A)  $3\log_4 x - 5\log_4 y + 2\log_4 z$

B)  $2\log x + \frac{1}{2}\log y$

C)  $\frac{1}{3}\log 6 + \frac{1}{3}\log x + \frac{2}{3}\log y$

D)  $\frac{3}{4}\log_3 16 - \frac{1}{3}\log_3 x^3 - 2\log_3 y$

E)  $3\log_5 x + 2\log_5 y + \log_5 z + 2$

F)  $\frac{1}{3}\log_2 x + \frac{2}{3}\log_2 y - 3$

G)  $\log_3(x+2) + \log_3(x-2) - \log_3(x+4)$

H)  $\frac{2}{3}\log(x+1) + \frac{1}{3}\log(x-2) - \frac{1}{3}\log(x+5)$

I)  $\frac{1}{3}[2\ln(x+3) + \ln x] - \ln(2x-1)$

J)  $\ln(x+3) - \ln(2x+5) + 2\ln(x-1)$

$$\mathbf{K)} \quad \frac{1}{2} [\ln(x+3) + 2\ln(x-1)] - 3\ln x$$

$$\mathbf{L)} \quad 2\ln 3 + 6\ln x - \frac{2}{3}\ln 27$$

$$\mathbf{M)} \quad \frac{1 + \log_3 x}{2}$$

$$\mathbf{N)} \quad \ln x - 2[\ln(x+2) + \ln(x-2)]$$

### Practice Using Properties of Logarithms

Use the following information, to approximate the logarithm to 4 significant digits by using the properties of logarithms.

$$\log_a 2 \approx 0.3562, \quad \log_a 3 \approx 0.5646, \quad \text{and} \quad \log_a 5 \approx 0.8271$$

$$\mathbf{A)} \quad \log_a \frac{6}{5}$$

$$\mathbf{B)} \quad \log_a 18$$

$$\mathbf{C)} \quad \log_a 100$$

$$\mathbf{D)} \quad \log_a 30$$

$$\mathbf{E)} \quad \log_a \sqrt{3}$$

$$\mathbf{F)} \quad \log_a \sqrt{75}$$

$$\mathbf{G)} \quad \log_a \frac{4}{9}$$

$$\mathbf{H)} \quad \log_a \sqrt[3]{15}$$

$$\mathbf{I)} \quad \log_a 54^2$$



Use the following information for letters J – R.

$$\log_{10} 2 \approx 0.3010, \quad \log_{10} 3 \approx 0.4771, \quad \text{and} \quad \log_{10} 7 \approx 0.8451$$

**J)**  $\log_{10} 28$

**K)**  $\log_{10} 8$

**L)**  $\log_{10} 4.5$

**M)**  $\log_{10} \sqrt{98}$

**N)**  $\log_{10} 20$

**O)**  $\log_{10} 210$

**P)**  $\log_{10} \frac{1}{300}$

**Q)**  $\log_{10} \frac{1}{9}$

**R)**  $\log_{10} \sqrt{42}$

## Base Change Formula

*Up to this point calculators have not been used to evaluate logarithms. Remember, the logs on all calculators are base 10. If a calculator is used to evaluate a log with any other base, it will not give the correct estimate unless a specific formula is used. The following base change formula will be used to evaluate any log with a base other than 10.*

$$\log_a b = \frac{\log b}{\log a}$$

*The terms  $\log a$  and  $\log b$  will be evaluated with a base 10, which means a calculator can now be used. Of course natural logs are always base  $e$ , so there is no need for a formula with natural logs. Each calculator is different. Be aware of what keystrokes are needed for your particular calculator, whether the log must be entered first, or the number first. Be very careful about grouping symbols when entering these types of problems into the calculator.*

Using a calculator, evaluate each of the following. Round all answers to three decimal places.

- |                          |                   |                         |               |
|--------------------------|-------------------|-------------------------|---------------|
| A) $\log_3 12$           | B) $\log_6 17$    | C) $\log_3 \frac{1}{5}$ | D) $\log_4 8$ |
| E) $\log_6 \frac{1}{12}$ | F) $\log_7 (-35)$ | G) $\ln 14$             | H) $\ln 3.26$ |
| I) $\ln \frac{1}{2}$     | J) $\ln 0$        | K) $\ln e$              | L) $\ln 6.2$  |

Why can't you take the log of a negative number?

## Solving Logarithmic Equations

*Here we will solve some logarithmic equations. There are a couple of ways to solve a logarithmic equation. One method is to get all logs to the left side of the equal sign and condense the problem so there is only one log. The next step would be to write the problem in exponential form and solve.*

*Another method involves one of the properties of simple logarithms. In particular, the property that states: “If  $\log_a x = \log_a y$  then  $x = y$ .” If the equation can be manipulated to resemble this property, take what is inside of each and set the two terms equal to each other. This follows the same principal as exponential equations where the bases are identical. If an exponential equation states  $2^{3x+4} = 2^7$ ; it stands to reason, because of the equal sign, that  $3x+4 = 7$ . The problem would then be solved from this point. With logarithmic equations, the problem  $\log_4(8x-5) = \log_4 12$  can be written as  $8x-5 = 12$ .*

*Always check for extraneous roots!!!*

**Solve each of the following logarithmic equations. (Round any solutions with decimals to three decimal places)**

**A)**  $\log_4 x + 6 = 8$

**B)**  $2\log_3 x - 4 = 1$

**C)**  $5 + 2\log_4 x = 4$

**D)**  $\log_3 x - \log_3 2 = 5$

**E)**  $\log(2x-1) = 0$

**F)**  $\log(4x+8) = 2$

**G)**  $\log_3(x+5) + \log_3(x+3) = \log_3 35$

**H)**  $\log(x+3) + \log(x-4) = \log 30$

**I)**  $\log_3 3x - 2 = 2$

**J)**  $\log_5 x + \log_5 (x+4) = 4\log_5 2$

**K)**  $\log_x 625 = 4$

**L)**  $\log_2 (x-2) + \log_2 (x-6) = \log_2 13$

**M)**  $2\log_3 x - \log_3 (x-2) = 2$

**N)**  $\ln x - \ln 2 = 0$

**O)**  $\ln(3x+5) = 8$

**P)**  $\ln(x-1) = 3.2$

**Q)**  $\log_3 (4x-2) = \log_3 (x+6)$

**R)**  $\ln(3x-2) = 4$

**S)**  $\ln e^{x+7} = 10$

**T)**  $\log_a (x^2 + 7) = \frac{2}{3} \log_a 64$

**U)**  $\log_2 (x+3) + \log_2 (x-3) = 4$

## Solving Exponential Equations

*You will now be solving exponential equations. Logarithms allow you to solve equations with variables in the exponent. We have dealt with these types of problems before, however, in the previous examples the bases of the problem could be made to match making solving the problem as simple as setting the two exponents equal to each other and solving for the variable. When ever you come to a problem with the variable in the exponent, and you cannot get the bases to match, you will need to solve the problem using logarithms. You will be using all three properties of logarithms to simplify and eventually solve the problem. You can take either the log or natural log of both sides, it makes no difference unless  $e$  is in the problem. In that case, you must use natural logs to solve the equation.*

**Solve each of the following exponential equations. Round solutions to three decimal places.**

A)  $3^{2x} = 5$

B)  $6^{4x} = 300$

C)  $12^{3x+1} = 7^2$

D)  $5^{x+2} = 3^5$

E)  $4^{5x+8} = 8^{x-1}$

F)  $9^{3x+2} = 27^{x+8}$

G)  $12^{3x-2} = 8^{5x+1}$

H)  $5^{6x-5} = 6^{2x+1}$

**I)**  $4e^{3x} = 40$

**J)**  $1200e^x = 900$

**K)**  $3e^{4x} = 12$

**L)**  $e^{2x} - 3e^x + 2 = 0$

**M)**  $e^{2x} - 5e^x + 6 = 0$

**N)**  $e^{2x} + 2e^x - 24 = 0$

**O)**  $2e^{2x} + e^x - 10 = 0$

**P)**  $5^{2x} - 7 \cdot 5^x + 10 = 0$

**Q)**  $3^{2x} + 5 \cdot 3^x - 14 = 0$

**R)**  $\log_8(\log_4(\log_6 x)) = \frac{1}{3}$

**S)**  $\log_2(\log_4 x) = 1$

**T)**  $\log_3(\log_{27} x) = -1$

### Finding the Domain of a Logarithmic Function

*Since we can't take the log of zero or the log of a negative number, we can find the domain of a logarithmic function using a simple inequality. Set what is inside the log greater than zero and solve. The result will give you the domain of the function. Consider the following.*

$$f_{(x)} = a \log_n (bx + c) + d$$

$$f_{(x)} = a \ln (bx + c) + d$$

*The variables  $a$ ,  $b$ ,  $c$  and  $d$  are used here to represent numbers that could possibly be used. To find the domain of either of these functions, evaluate the following inequality:  $bx + c > 0$ .*

**Find the domain of each of the following logarithmic functions. Be sure to write the domain using interval notation.**

**A)**  $f_{(x)} = \log_5 x + 2$

**B)**  $f_{(x)} = \log_3 (4x - 1) - 2$

**C)**  $f_{(x)} = -\log_2 3x$

**D)**  $f_{(x)} = \log_2 (5 - x)$

**E)**  $f_{(x)} = \log_5 (-x) + 5$

**F)**  $f_{(x)} = \ln x - 4$

**G)**  $f_{(x)} = \ln (2 - 3x)$

**H)**  $f_{(x)} = \log_3 (x + 5) + 1$

**I)**  $f_{(x)} = \ln (x - 3)$

**J)**  $f_{(x)} = \log_2 |x|$

**K)**  $f_{(x)} = |\log_3 x|$

**L)**  $f_{(x)} = -\ln 5x + 1$

**Explain the difference between the domains of letters J and K above. Although each problem contains an absolute value, the domains are different. Why is this?**

### Finding the Vertical Asymptote of a Logarithmic Function

$$f_{(x)} = a \log_n (bx + c) + d$$

$$f_{(x)} = a \ln (bx + c) + d$$

*To find the vertical asymptote of a logarithmic function, set  $bx+c$  equal to zero and solve. This will yield the equation of a vertical line. In this case, that vertical line is the vertical asymptote.*

#### *Example*

Find the vertical asymptote of the function  $f_{(x)} = \log_3 (4x - 3) - 2$ .

$$4x - 3 = 0$$

$$4x = 3$$

$$x = \frac{3}{4}$$

*Setting the  $bx+c$  term equal to zero results in this equation. Simply solve for  $x$ , and we now have the vertical asymptote of the function.*

Find the vertical asymptote of each of the following logarithmic functions.

A)  $f_{(x)} = \log_5 x + 2$

B)  $f_{(x)} = \log_3 (4x - 1) - 2$

C)  $f_{(x)} = -\log_2 3x$

D)  $f_{(x)} = \log_2 (5 - x)$

E)  $f_{(x)} = \log_5 (-x) + 5$

F)  $f_{(x)} = \ln x - 4$

G)  $f_{(x)} = \ln (2 - 3x)$

H)  $f_{(x)} = \log_3 (x + 5) + 1$

I)  $f_{(x)} = \ln (x - 3)$

J)  $f_{(x)} = \log_2 |x|$

K)  $f_{(x)} = |\log_3 x|$

L)  $f_{(x)} = -\ln 5x + 1$



## Graphing Logarithmic Functions

$$f_{(x)} = a \log_n (bx + c) + d$$

$$f_{(x)} = a \ln (bx + c) + d$$

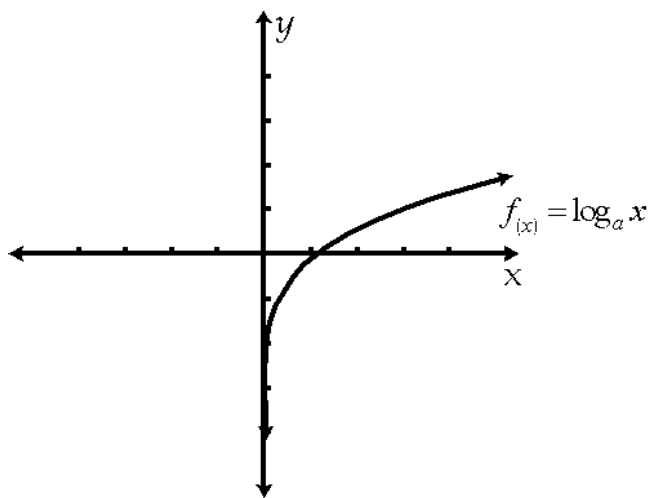
*The parent functions for both logarithmic and natural logarithmic functions look almost identical. The following functions will be graphed by setting up a table. When faced with the following function:  $f_{(x)} = \log_3 x$ , find the domain of the function first by evaluating*

*$bx + c > 0$ . This is done to ensure that only appropriate values are substituted for  $x$ . The values that should be used for  $x$  are powers of the base of the logarithm such as*

*$\frac{1}{9}$ ,  $\frac{1}{3}$ , 1, 3, and 9. The values used to substitute for  $x$  are completely dependant on the*

*base of the particular logarithm in the problem. When graphing a natural log with a table, any number in the domain of the function may be used. Using a calculator, get the decimal estimates for the  $y$  values of the function. DO NOT FORGET to plug in fractional powers for both logs and natural logs. The fractional powers will give the tail end of the function. Be sure to plug in the fractions that are right next to the vertical asymptote of the function. Note in the following picture the graph does not cross the  $y$  axis. That is because, in this particular example,  $x = 0$  is the vertical asymptote. By now, you should be aware that the equation  $x = 0$  is the equation for the  $y$  axis. Remember, always find the vertical asymptote of any logarithmic function when graphing. This must be done to ensure that it is not crossed. The range of any normal logarithmic function is all real numbers. The only time this will not be is the case is that of example K in the finding the domain section on the previous page.*

*Remember, most people forget the tail end of the function. Do not forget it. Make sure you note the rise of the function. The function makes a very slow gradual rise. It is not steep.*

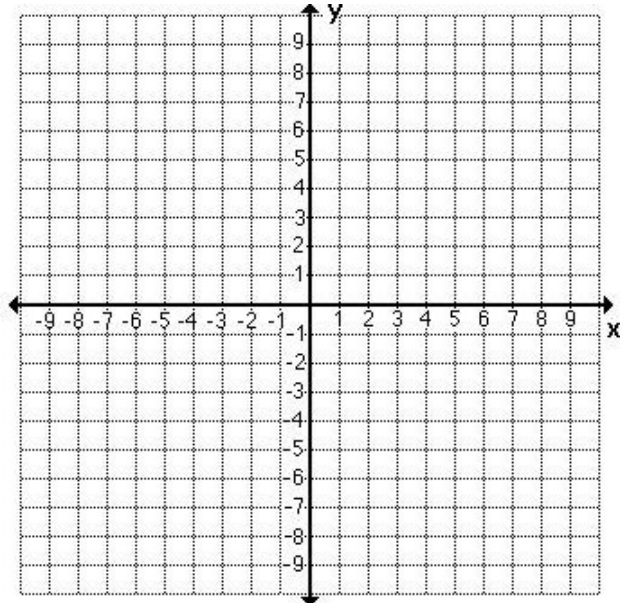


*Normally, we will be graphing these functions by means of translations based on the parent function. This graphing portion is meant to give you practice at using your knowledge about logarithms and how to use powers to graph the functions. There will be an entire section devoted to nothing but functions later on in the workbook.*

**Graph each of the following by setting up a table. Identify the range and domain of each function. Label the x intercept of each function.** *We will deal with y intercepts later.*

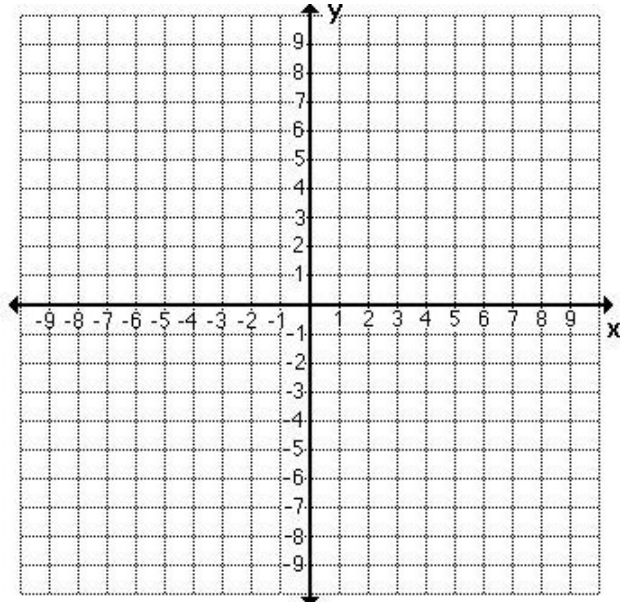
A)  $f_{(x)} = \log_3 x$

$x$	$f_{(x)}$



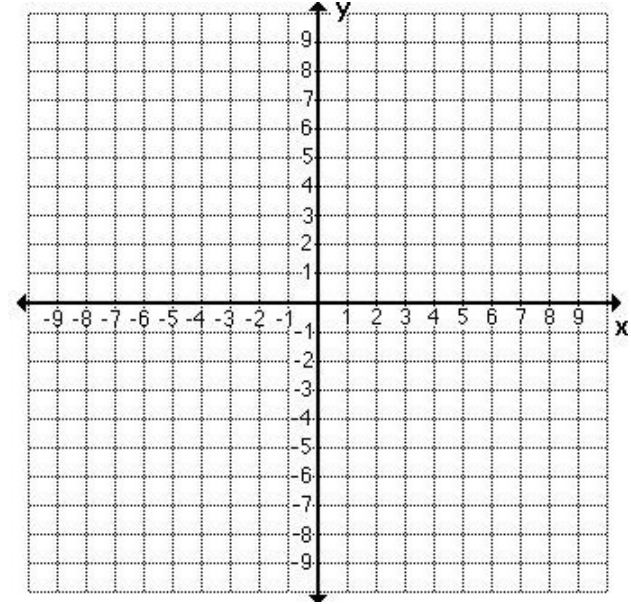
B)  $f_{(x)} = \log_4 x - 2$

$x$	$f_{(x)}$



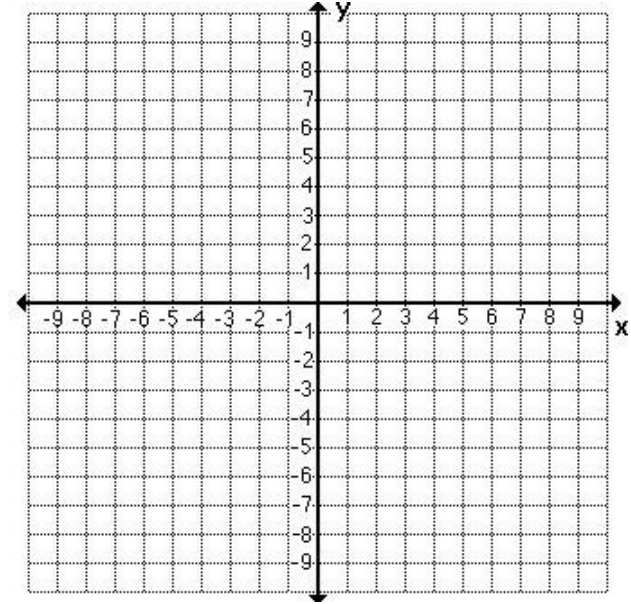
C)  $f_{(x)} = -\log_2 x$

$x$	$f_{(x)}$



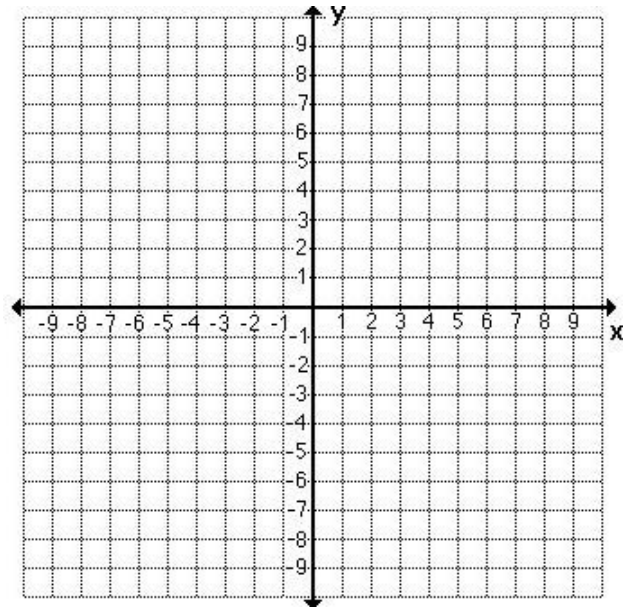
D)  $f_{(x)} = \log_3 (x+1)$

$x$	$f_{(x)}$



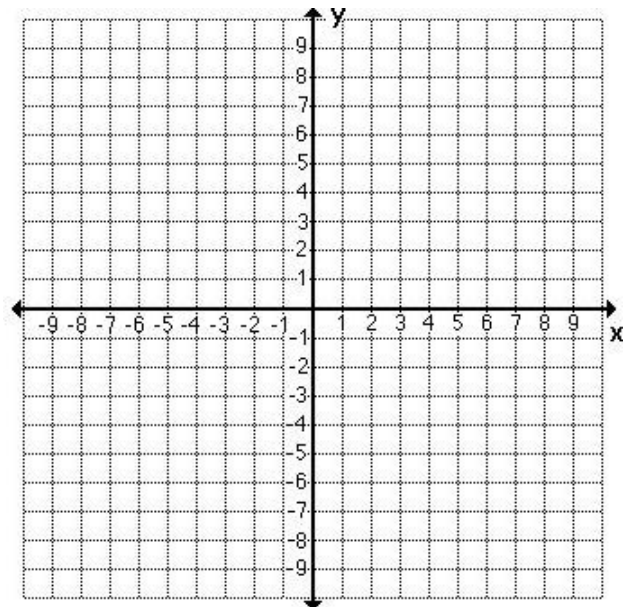
**E)**  $f(x) = \ln x + 3$

$x$	$f(x)$
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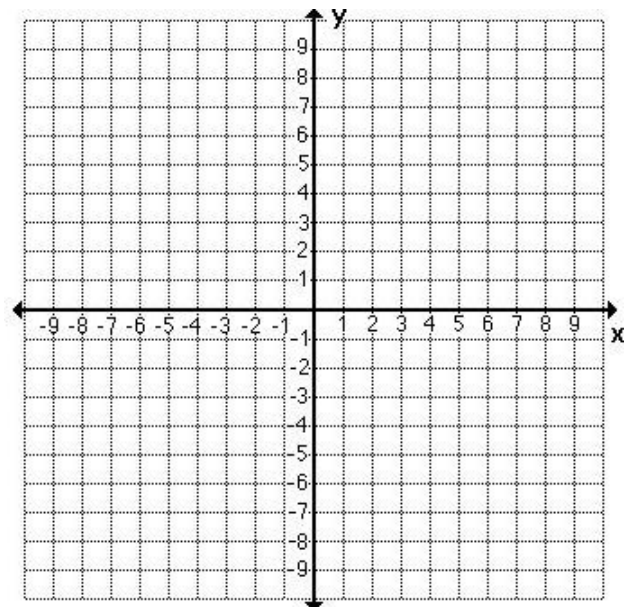
**F)**  $f(x) = \ln(x+2)$

$x$	$f(x)$
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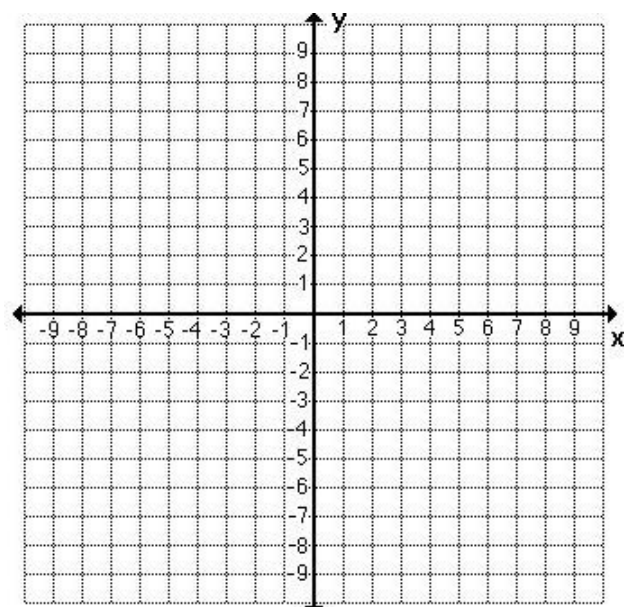
**G)**  $f_{(x)} = \log_3(-x)$

$x$	$f_{(x)}$
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**H)**  $f_{(x)} = |\log_2 x|$

$x$	$f_{(x)}$
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## Finding the Inverse of a Function

*In the past we worked on “function operations,” in which we added, subtracted, multiplied, divided, and found composite functions (refer to page 68). The next function operation we will go over is finding the inverse of a function. If a function is denoted by  $f_{(x)}$ , its’ inverse is denoted by  $f_{(x)}^{-1}$ . The “-1” is a superscript of the term, not an exponent.*

**Finding the inverse of a function is a 4 step process.**

1. Replace  $f_{(x)}$  with  $y$
2. Switch the  $x$  and  $y$  in the problem.
3. Solve for  $y$
4. Replace  $y$  with  $f_{(x)}^{-1}$

*A function only has an inverse if it is a one-to-one function. This means that for each  $y$  value, there exists only one corresponding  $x$  value. Simply put, a function is one-to-one if it passes a horizontal line test.*

*Proving whether or not two functions are inverses of each other is a different matter. Two functions  $f_{(x)}$  and  $g_{(x)}$  are inverses of each other if and only if the following is true:*

$$f_{(g_{(x)})} = x \text{ and } g_{(f_{(x)})} = x$$

*If **both** composite functions  $f_{(g_{(x)})}$  and  $g_{(f_{(x)})}$  equal  $x$ , the functions are inverses of each other.*

*The official definition states:  $f_{(g_{(x)})} = x$  for all  $x$  in the domain of  $g$ .*

$$g_{(f_{(x)})} = x \text{ for all } x \text{ in the domain of } f.$$

**Here is an example of finding the inverse of an exponential function.**

$$f_{(x)} = 3^x - 1$$

$$y = 3^x - 1 \quad \text{Replace } f_{(x)} \text{ with } y.$$

$$x = 3^y - 1 \quad \text{Switch the } x \text{ and } y.$$

$$x + 1 = 3^y \quad \text{Solve for } y \text{ by adding } 1 \text{ to both sides.}$$

$$\log(x + 1) = \log 3^y \quad \text{Now you need to get to the } y \text{ exponent, so take the log of both sides.}$$

$$\log(x + 1) = y \log 3 \quad \text{Pull out the exponent } y \text{ using the properties of logarithms.}$$

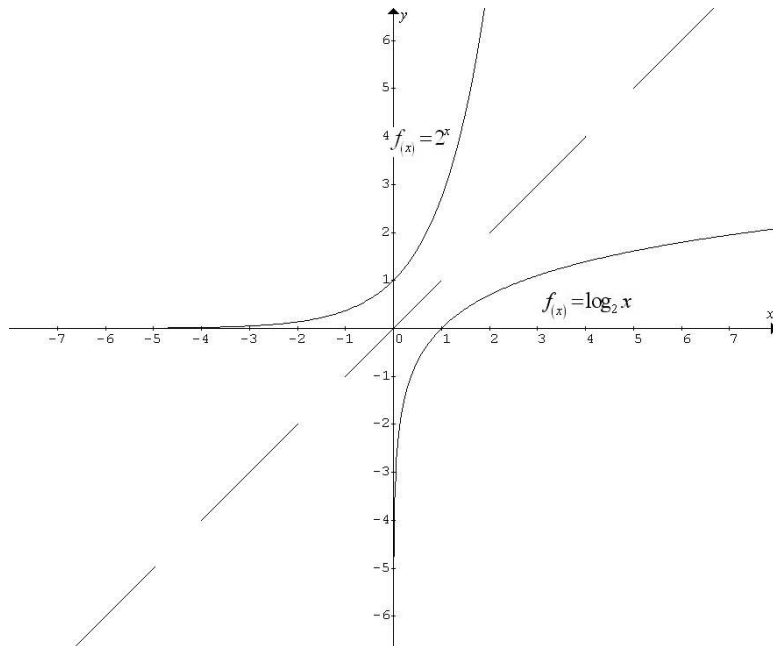
$$\frac{\log(x + 1)}{\log 3} = y \quad \text{Divide both sides now by } \log 3.$$

$$y = \log_3(x + 1) \quad \begin{array}{l} \text{Use the symmetrical property to switch sides.} \\ \text{Using the base change formula, you can rewrite the log with a base of } 3. \end{array}$$

$$f_{(x)}^{-1} = \log_3(x + 1) \quad \text{Replace } y \text{ with } f_{(x)}^{-1}$$

*As demonstrated in the previous example, the inverse of an exponential function is a logarithmic function and vice-versa.*

*Inverse functions are symmetrical to the  $y = x$  axis. When an exponential function and its' logarithmic inverse are graphed on the same plane, the symmetry is apparent.*



*Drawing an imaginary diagonal using the line  $y = x$ , the functions could literally be folded along that axis, and the two functions would line up on top of each other.*

*In the following exercises you will be asked to find the inverse of a logarithmic function. It should be obvious that the inverse of a logarithmic function is exponential. In order to get to that point, the problem will need to be rewritten in exponential form.*

**Logarithmic form**

$$\log_a b = c$$

**Exponential Form**

$$a^c = b$$

**Here is an example of finding the inverse of a logarithmic function.**

$$f_{(x)} = \log_7(x - 2) + 3$$

$$y = \log_7(x - 2) + 3$$

*Replace  $f_{(x)}$  with  $y$ .*

$$x = \log_7(y - 2) + 3$$

*Switch the  $x$  and  $y$ .*

$$x - 3 = \log_7(y - 2)$$

*Isolate the log by subtracting 3 to both sides.*

$$\log_7(y - 2) = x - 3$$

*Using the symmetrical property, the log is easier to see like this.*

$$7^{x-3} = y - 2$$

*Rewrite the log in exponential form.*

$$7^{x-3} + 2 = y$$

*Solve for  $y$  by adding 2 to both sides.*

$$y = 7^{x-3} + 2$$

*Again rewrite using the symmetrical property.*

$$f_{(x)}^{-1} = 7^{x-3} + 2$$

*Replace  $y$  with  $f_{(x)}^{-1}$*

Find the inverse of each of the following functions if it exists.

**A)**  $f_{(x)} = \frac{1}{2}x + 3$

**B)**  $f_{(x)} = 4x - 5$

**C)**  $f_{(x)} = 4^{x+2}$

**D)**  $f_{(x)} = 2^{x-3} + 1$

**E)**  $f_{(x)} = \sqrt[3]{3x-2}$

**F)**  $f_{(x)} = 4(x+1)^2 - 3$

**G)**  $f_{(x)} = \log_6 x$

**H)**  $f_{(x)} = \ln(x-2)$

**I)**  $f_{(x)} = 2\log_4 x - 5$

**J)**  $f_{(x)} = -\sqrt{x-4} + 6$

**K)**  $f_{(x)} = (x+3)^3 - 2$

**L)**  $f_{(x)} = 3^{x-20} + 1$



## Interest Formulas

*There are three interest formulas commonly used in mathematics. The first is the standard interest formula. We will not be using that formula in this section. Instead, we will concentrate on the other two. We will be working with the compound interest formulas. There are two formulas for this. The first is a formula for interest being compounded a finite number of times per year. The second formula is for interest compounded continuously. For each of these functions, the variable time,  $t$ , should be in years. Which means 6 months is .5 years.*

*Standard Interest  
Formula*

$$A = P + Prt$$

*Compound Interest Formula*

$$A = P \left( 1 + \frac{r}{n} \right)^{nt}$$

*Interest Compounded  
continuously*

$$A = Pe^{rt}$$

**When using the compound interest formula, what does the variable  $n$  represent?**

**When dealing with the compound interest formula, your questions will always come in the form of word problems. Write down the word associated with the number of times interest is to be compounded per year when dealing with these word problems.**

<i># of times compounded</i>	<i>Word used in the problem</i>
<b>once</b>	
<b>twice</b>	
<b>four times</b>	
<b>twelve times</b>	
<b>twenty-four times</b>	

**Explain, in your own words, what compounding interest means.**

**If you had a choice to invest a sum of money into an account that yields 7% interest compounded quarterly versus one that compounds interest semi-annually, which would you choose?**

**Answer each of the following.**

- A) If you invest \$2500 in an account that pays 12% interest, compounded quarterly, how much would you have at the end of 17 years?
- B) How much would you have to invest in an account that pays 6% interest, compounded monthly, to have a balance of \$30,000 at the end of 10 years?
- C) How long will it take for an investment of \$2,000 in an account that pays  $8\frac{1}{2}\%$  interest compounded quarterly to become \$15,000.
- D) How long will it take for an amount of money to double if deposited in an account that pays 4.5% interest compounded monthly?

**E)** At what interest rate must you invest \$10,000 to have an ending balance of \$72,000 at the end of 14 years? (Assume interest is compounded quarterly.)

**F)** If you invest \$26,000 in an account that pays  $3\frac{1}{2}\%$  interest compounded continuously, how much would your investment be worth in 18 years.

**G)** How long will it take for an amount of money to double if invested in an account that pays 9% interest compounded continuously.

**H)** At what interest rate must you invest \$10,000 to have an ending balance of \$15,000 at the end of 9 years? (Assume interest is compounded continuously.)

**I)** How much must your principle investment be if you have \$30,000 after 16 years in an account that pays 7.5% interest compounded continuously.

- J)** Complete the following table to determine the value,  $A$ , of an initial investment,  $P$ , of \$15,000 into an account that pays 6.5% interest compounded  $n$  times for 10 years.

<b>n</b>	<b>1</b>	<b>2</b>	<b>4</b>	<b>12</b>	<b>365</b>
<b>A</b>					

- K)** Complete the table for the time necessary for an account to triple if interest is compounded continuously at rate  $r$ .

<b>r</b>	<b>2%</b>	<b>3%</b>	<b>4.5%</b>	<b>7%</b>	<b>12%</b>
<b>t</b>					

### Word Problems

A) The demand equation for a certain clock radio is given by  $p = 400 - .06e^{0.003x}$ . Find the demand,  $x$ , for the price of  $p = \$99$ .

B) The population,  $P$ , where  $P$  is measured in thousands, of one city is given by  $P = 30e^{kt}$ . In this particular model,  $t = 0$  represents the year 2000. In 1990, the population was 52,000. Find the value of  $k$  and use the result to estimate the population of the city in the year 2012.

*On the Richter scale, the magnitude  $R$  of an earthquake with intensity  $I$  is measured by*

$$R = \log_{10} \frac{I}{I_0}$$

*Where  $I_0 = 1$  is the minimum intensity used for comparison.*

C) Find the intensity of an earthquake that measures 6.5 on the Richter scale.

D) Find the intensity of an earthquake the measures 3.2 on the Richter scale.

E) Find the magnitude of an earthquake that has an intensity of 325,000.

## Checking Progress

*You have now completed the “Logarithms” section of the workbook. The following is a checklist so that you may check your progress. Check off each of the objectives you have accomplished.*

*The student should be able to...*

- Evaluate a simple logarithm without the aid of a calculator.*
- Express a logarithmic statement in exponential form.*
- Express a statement in exponential form in logarithmic form*
- Expand a logarithmic expression as the sum or difference of logarithms using the properties of logs.*
- Condense the sum or difference of logarithms into a single logarithmic expression.*
- Evaluate logarithms using the base change formula.*
- Solve logarithmic equations.*
- Evaluate the solution to logarithmic equations to find extraneous roots.*
- Solve equations with variables in the exponents.*
- Find the range and domain of logarithmic functions.*
- Graph a logarithmic function using a table.*
- Find the inverse of a function.*
- Verify two functions are inverses of each other.*
- Identify a one-to-one function.*
- Use the compound interest formulas.*