Radical Functions

For radical functions we will use the equation $f(x)=a\sqrt{x-h}+k$ to denote the standard form of the equation. Be aware, that the variable x may have a coefficient from time to time. Follow the standard procedure to find the x and y intercepts of any radical function. Set the x or y equal to zero, depending on which one you wish to find, and solve for the remaining variable. Finding the domain of a radical function is a little tricky. To find the domain of any radical function <u>with an even index</u>, set the radicand greater that or equal to zero ($\geq$) and solve. If the radicand is a polynomial, you will need to solve the polynomial inequality by finding critical points, and testing intervals. To find the range of the radical function, find y value of the point of origin, and use the constant a to determine the range of the function.

Given the radical function $f(x)=-\sqrt{x+4}-3$, the following can be determined.

First find the domain of the function. This will give you the x value needed for the point of origin.

$$f(x)=-\sqrt{x+4}-3$$

Finding the domain.

$$x+4\geq 0$$

$$x\geq -4$$

You can see the domain of the function is $[-4,\infty)$.
The -4 is the x value the point of origin.

Finding the range.

Since the constant a is -1, the function will go downwards. Meaning that the range is $(-\infty,-3]$.

Finding the "point of origin" of a radical function.

To find the point of origin of a radical function use the rules discussed in previous sections. The point of origin for the parent function $y=\sqrt{x}$ is (0, 0). This particular graph will shift left 4 and down 3, so the point of origin is $(-4,-3)$. Be careful when using these rules. Make sure to find the domain of the function before you attempt to find the point of origin. Consider a function such as $y=\sqrt{3-x}$. Since there is a positive 3 inside the radicand, you would normally shift to the left 3. However, If you were to find the domain of this function by setting the radicand ≥ 0, You will find the domain is actually $x\leq 3$. This says the graph is shifting to the right 3 spaces.

Finding the x-intercept.

Substitute 0 for y and solve for x.

$$0=-\sqrt{x+4}-3$$

$$\sqrt{x+4}=-3$$

This is not possible. That means there is no x intercept for this function.

Finding the y-intercept.

Substitute 0 for x and solve for y.

$$y=-\sqrt{(0)+4}-3$$

$$y=-\sqrt{4}-3$$

$$y=-2-3$$

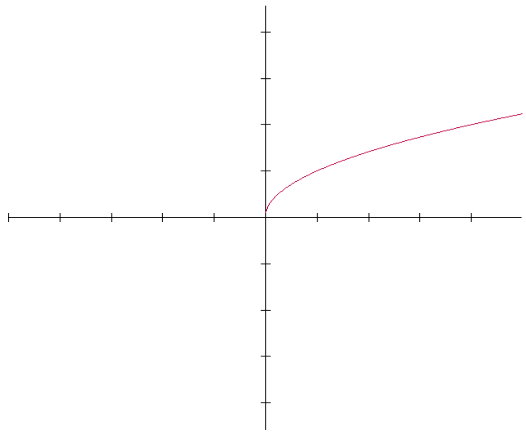
$$y=-5$$

The y intercept of this function is (0, -5).

We will now look at the parent function, and some translations of it.

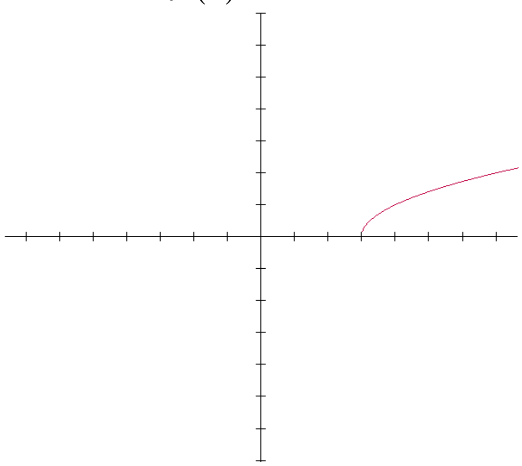
$$f(x) = a\sqrt{x-h} + k$$

$$f(x) = \sqrt{x}$$



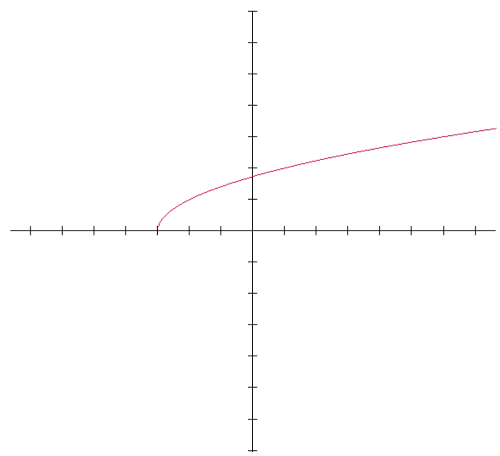
The parent function has the point of origin at (0, 0)

$$f(x) = \sqrt{x-3}$$



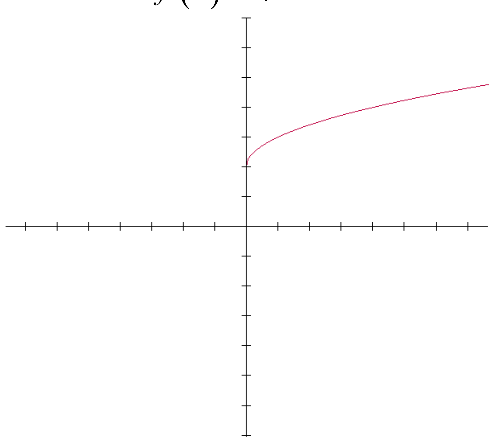
The graph of this function shifts right 3.

$$f(x) = \sqrt{x+3}$$



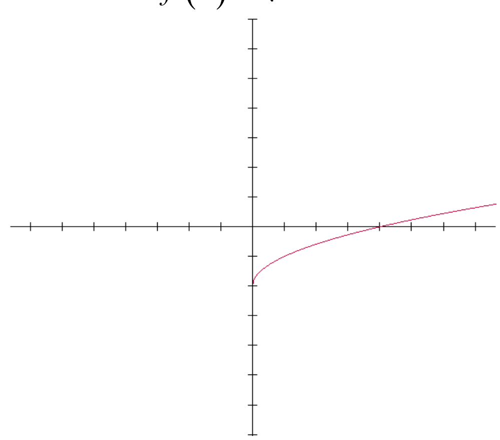
The graph of this function shifts left 3.

$$f(x) = \sqrt{x} + 2$$



Here the graph shifts up 2.

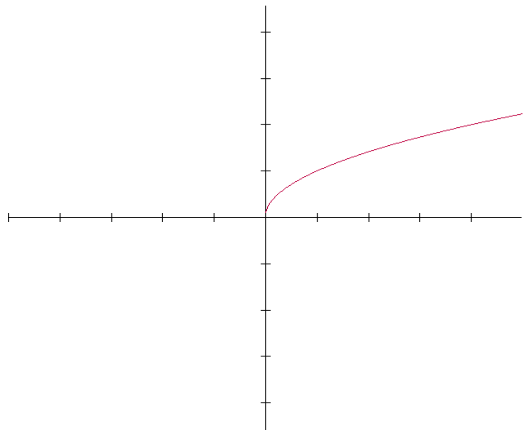
$$f(x) = \sqrt{x} - 2$$



The graph of this function shifts down 2.

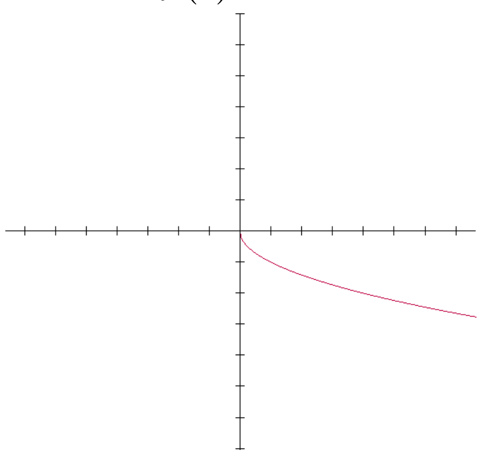
$$f(x) = a\sqrt{x-h} + k$$

$$f(x) = \sqrt{x}$$



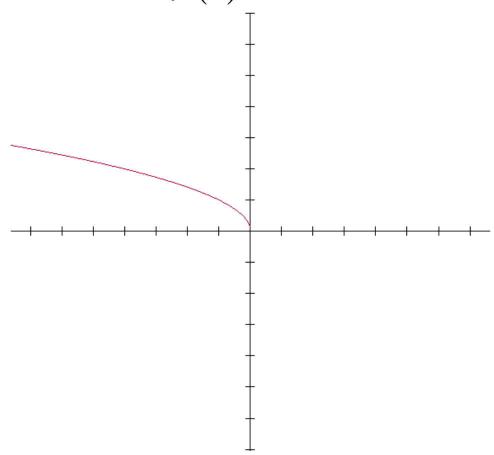
The parent function has the point of origin at (0, 0)

$$f(x) = -\sqrt{x}$$



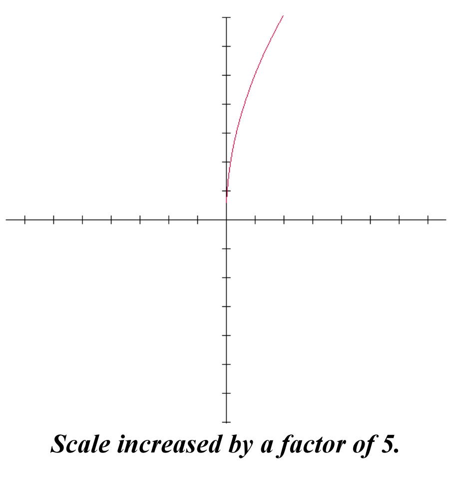
The graph of this function flips upside down.

$$f(x) = \sqrt{-x}$$



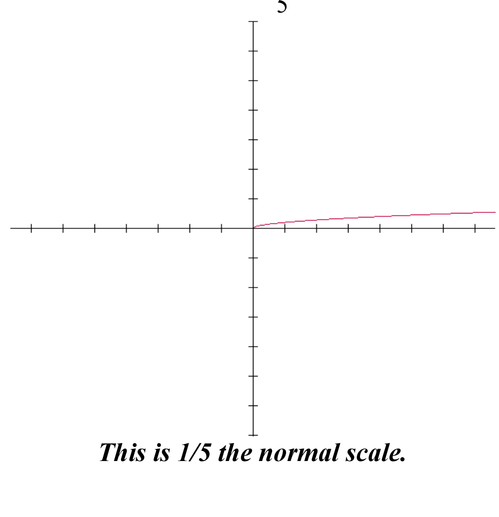
The graph of this function flips from right to left as the –x affects the domain of the function.

$$f(x) = 5\sqrt{x}$$



Scale increased by a factor of 5.

$$f(x) = \frac{1}{5}\sqrt{x}$$



This is 1/5 the normal scale.

Find the domain of each of the following radical functions in interval notation.

A) $f(x)=\sqrt{x+4}-2$

B) $f(x)=2\sqrt{4-x}+1$

C) $f(x)=\sqrt{2x+3}+1$

D) $f(x)=\sqrt{x^2-4}$

E) $f(x)=\sqrt{x^2}$

F) $f(x)=\frac{1}{2}\sqrt{6-x}-3$

G) $f(x)=-\sqrt{x+5}-8$

H) $f(x)=\sqrt{2-x}+1$

I) $f(x)=2\sqrt{x+7}-5$

The range of a radical function in $f(x)=a\sqrt{x-h}+k$ ***form can be found using the value of the "*** a ***" term, and the y value of the point of origin.***

If $a>0$***, the range of the function is*** $[k,\infty)$***.***

If $a<0$***, the range of the function is*** $(-\infty,k]$***.***

Find the range for each of the following.

A) $f(x)=\sqrt{x+5}-3$

B) $f(x)=-\sqrt{x-3}+2$

C) $f(x)=2\sqrt{x-4}+3$

D) $f(x)=-3\sqrt{5-x}+6$

E) $f(x)=\sqrt{4-x}-3$

F) $f(x)=\sqrt{x-7}+5$

Find the point of origin for each of the following radical functions.

A) $f(x)=\sqrt{x+4}-2$

B) $f(x)=2\sqrt{4-x}+1$

C) $f(x)=\sqrt{x}-4$

D) $f(x)=-\sqrt{x-3}$

E) $f(x)=\sqrt{x^2}$

F) $f(x)=\frac{1}{2}\sqrt{6-x}-3$

G) $f(x)=-\sqrt{x+5}-8$

H) $f(x)=\sqrt{2-x}+1$

I) $f(x)=2\sqrt{x+7}-5$

Why is the graph of the function $f(x)=\sqrt{-x}$ moving towards the left rather than the right?

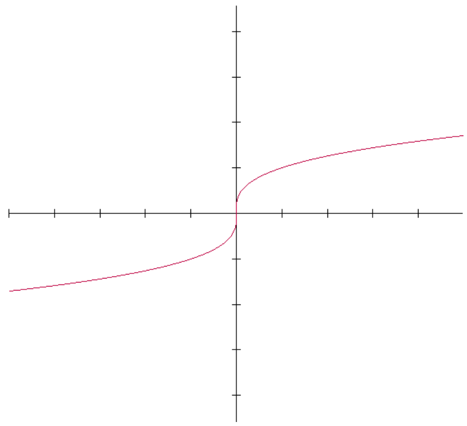
Explain why the graph of the function $f(x)=\sqrt{x^2}$ is identical to that of $f(x)=|x|$.

To find the domain of a radical function that has an even index, why do you need to set the radicand ≥ 0?

We will now look at the cube root function.

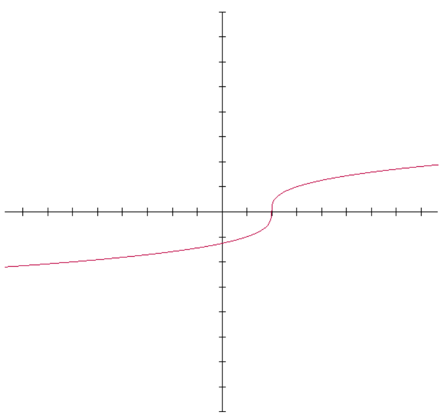
$$f(x) = a\sqrt[3]{x-h} + k$$

$$f(x) = \sqrt[3]{x}$$



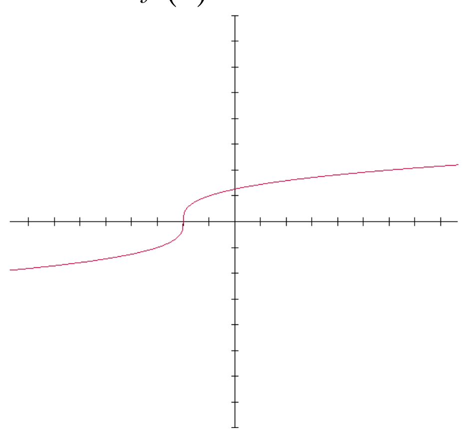
The parent function has the point of origin at (0, 0)

$$f(x) = \sqrt[3]{x-2}$$



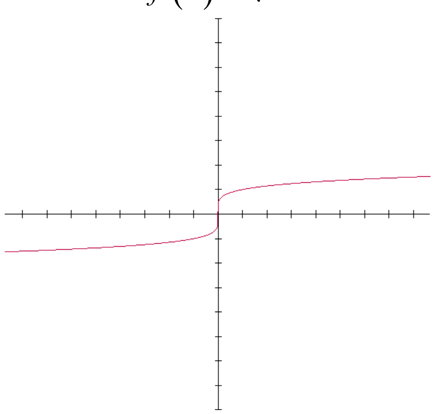
The graph of this function shifts right 2.

$$f(x) = \sqrt[3]{x+2}$$



The graph of this function shifts left 2.

$$f(x) = \sqrt[5]{x}$$

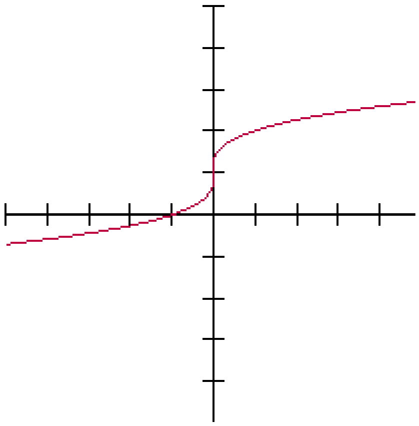


As you can see on the left, the curve is just about the same for a 5th root, verses a cubed root. This will be the same case for any radical function where the index is odd. This also means that any radical function where the index is even will look like a normal square root function. The curves of these functions are a little "flatter" than a regular square root or cubed root.

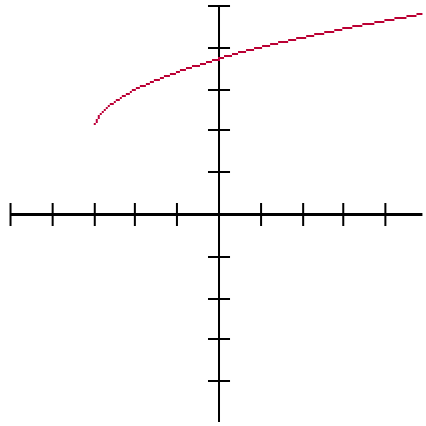
Vertical translations of the function are identical to that of a regular square root function. As you can see, the domain and range of any radical function with an odd index is all real numbers.

Match the appropriate graph with its equation below. Explain why each of your solutions is true.

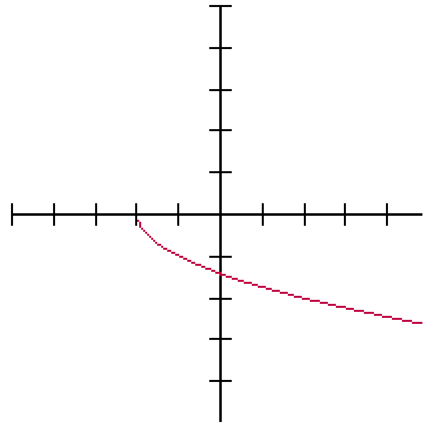
A



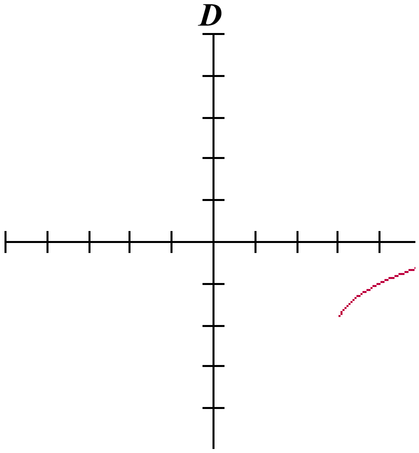
B



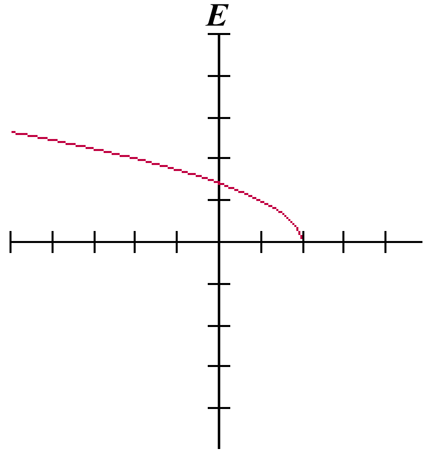
C



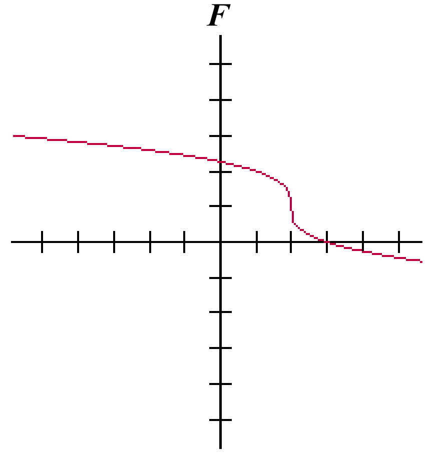
D



E



F



1) $f(x) = \sqrt{x+3} + 2$

2) $f(x) = \sqrt{x-3} - 2$

3) $f(x) = \sqrt[3]{x} + 1$

4) $f(x) = -\sqrt[3]{x-2} + 1$

5) $f(x) = -\sqrt{x+2}$

6) $f(x) = \sqrt{2-x}$

<u>**Getting a Precise Curve**</u>

Make sure the curve of the radical function is accurate. To ensure this, plot points by moving in perfect squares; beginning from the point of origin.

When graphing the parent function,

$$f(x)=\sqrt{x}$$

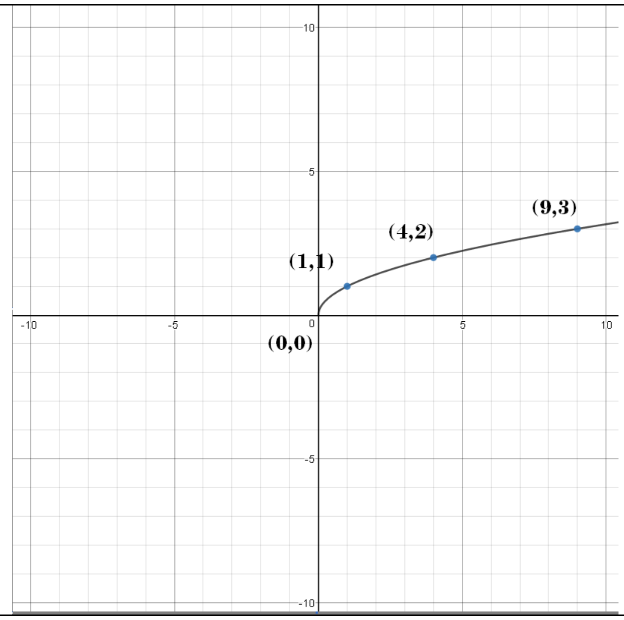
Begin at the point of origin, (0,0).

The $\sqrt{1}=1$***, so move right 1, up 1.***

The $\sqrt{4}=2$***, so move right 4, up 2.***

The $\sqrt{9}=3$***, so move right 9, up 3.***

If the graph allowed, to find the next point move right 16, up 4 from the point of origin. The x values are all perfect squares giving y values that are integers and easy to plot.



In the above example, the function f **is defined by** $f(x)=\sqrt{x}$**. This provides the algorithm, or set of instructions, necessary to find coordinates on the graph of function** f**. This was done in the past using tables of values and substitution.**

x	$f(x)$
0	0
1	1
4	2
9	3
16	4
25	5

The table to the left would be completed using the rule $f(x)=\sqrt{x}$

This results in the following.

$f(0)=\sqrt{0}$ $\quad f(1)=\sqrt{1}$ $\quad f(4)=\sqrt{4}$

$f(0)=0$ $\quad f(1)=1$ $\quad f(4)=2$

$f(9)=\sqrt{9}$ $\quad f(16)=\sqrt{16}$ $\quad f(25)=\sqrt{25}$

$f(9)=3$ $\quad f(16)=4$ $\quad f(25)=5$

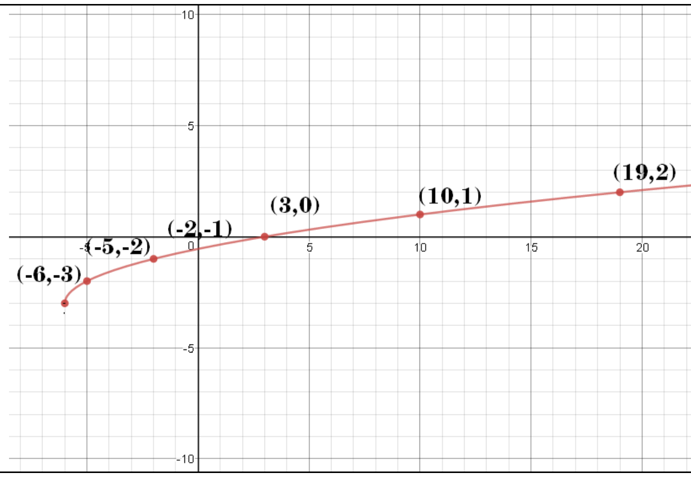
The same coordinates were found by counting and moving in terms of perfect squares.

Creating a table that yield integer coordinates gets more difficult when transformations are involved. Consider the function: $f(x) = \sqrt{x+6} - 3$

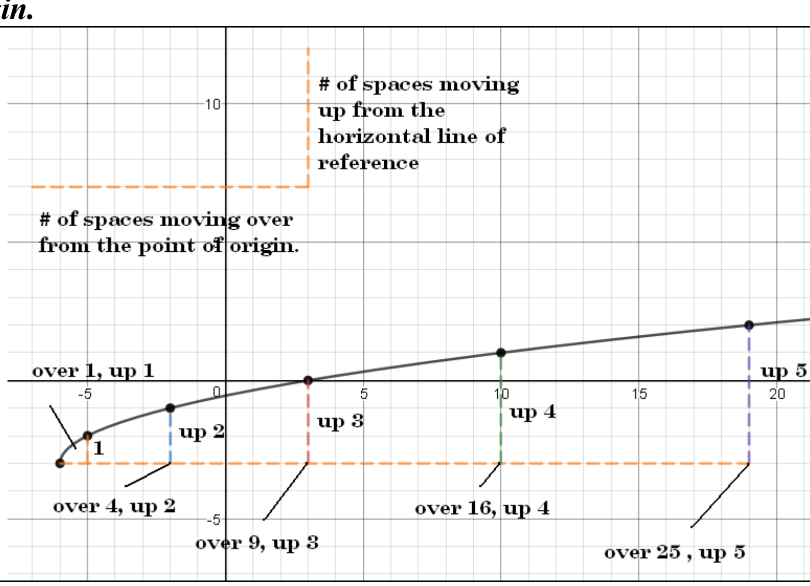
The perfect squares are 1, 4, 9, 16, 25, 36, 49 … and so on. To get a table that yields integer coordinates, the expression $x+6$ **needs to be taken into consideration.**

For the table, it is best to use values for x that yield a perfect square. As a result, the question must be answered…

Perfect square needed	Question?	Answer	New table x	New table $f(x)$
0	What number + 6 equals 0?	-6	**-6**	**-3**
1	What number + 6 equals 1?	-5	**-5**	**-2**
4	What number + 6 equals 4?	-2	**-2**	**-1**
9	What number + 6 equals 9?	3	**3**	**0**
16	What number + 6 equals 16?	10	**10**	**1**
25	What number + 6 equals 25?	19	**19**	**2**
36	What number + 6 equals 36?	30	**30**	**3**
49	What number + 6 equals 49?	43	**43**	**4**

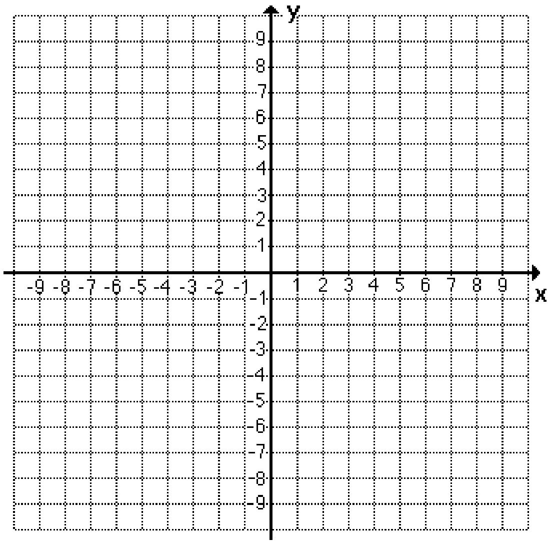


As the graph below illustrates, its much easier to find the coordinates by counting spaces from the point of origin.



Graph each of the following radical functions. ***Find all required information.***

A) $f(x)=\sqrt{x-3}+2$



Point of Origin:

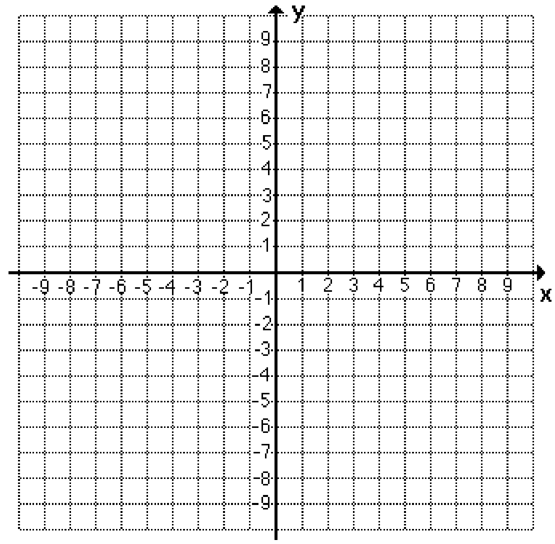
Y-intercept:

Range:

X-intercepts:

Domain:

B) $f(x)=-\sqrt{x-3}+1$



Point of Origin:

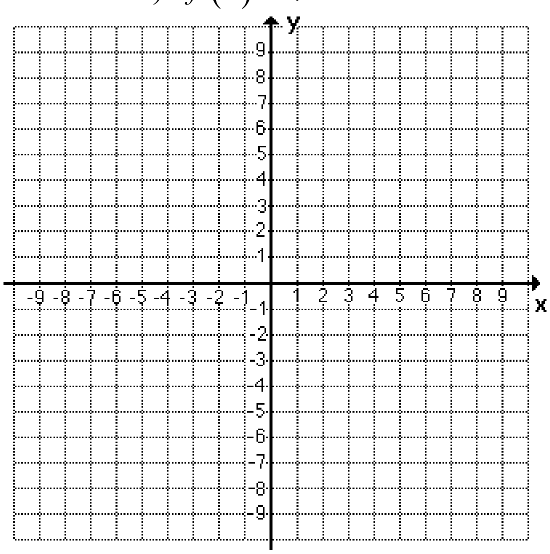
Y-intercept:

Range:

X-intercepts:

Domain:

C) $f(x)=\sqrt{3-x}+1$



Point of Origin:

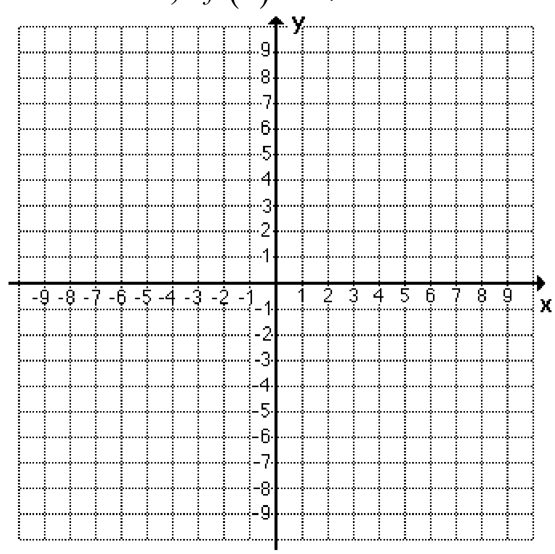
Y-intercept:

Range:

X-intercepts:

Domain:

D) $f(x)=2\sqrt{x}-4$



Point of Origin:

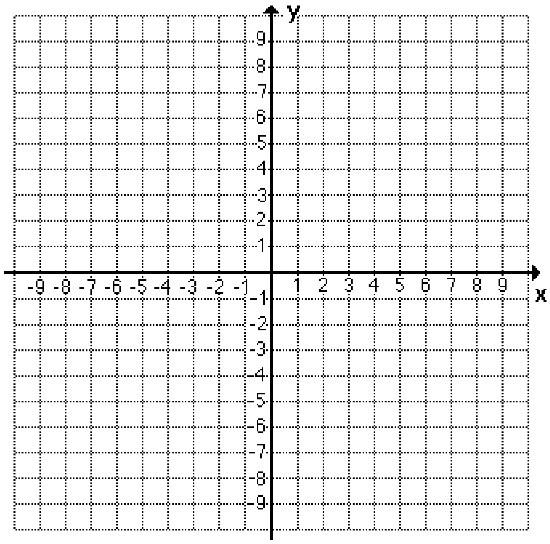
Y-intercept:

Range:

X-intercepts:

Domain:

E) $f(x) = -\sqrt{-x}$



Point of Origin:

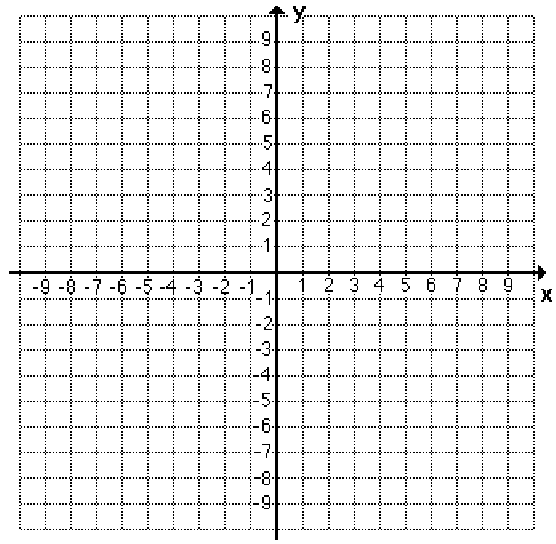
Y-intercept:

Range:

X-intercepts:

Domain:

F) $f(x) = \sqrt[3]{x+2} + 3$



Point of Origin:

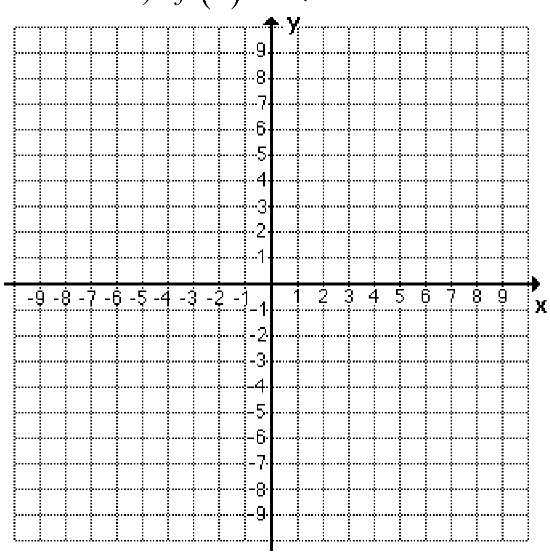
Y-intercept:

Range:

X-intercepts:

Domain:

G) $f(x) = -\sqrt[3]{x-3} - 2$



Point of Origin:

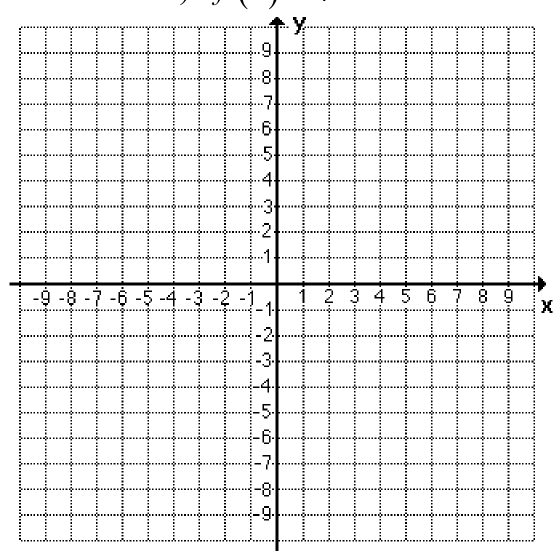
Y-intercept:

Range:

X-intercepts:

Domain:

H) $f(x) = \sqrt[3]{x-6}$



Point of Origin:

Y-intercept:

Range:

X-intercepts:

Domain:

Why are the graphs of $y = \sqrt[3]{x}$ and $y = -\sqrt[3]{-x}$ identical?