

Finding all Zeros of a Polynomial Function

When solving polynomial equations, use the rational zero test to find all possible rational zeros, then use Descarte's Rule of Signs to help narrow down the choices if possible. The fundamental theorem of Algebra plays a major role in this.

The Fundamental Theorem of Algebra

Every polynomial equation of degree n with complex coefficients has n roots in the complex numbers.

In other words, if you have a 5th degree polynomial equation, it has 5 roots.

Example: Find all zeros of the polynomial function $f_{(x)} = 2x^4 + 7x^3 - 4x^2 - 27x - 18$.

$$2x^4 + 7x^3 - 4x^2 - 27x - 18 = 0$$

Find all possible rational zeros.

$$\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}$$

For this equation, there is 1 possible positive zero, and either 3 or 1 possible negative zeros.

Now set up a synthetic division problem, and begin checking each zero until a root of the equation is found..

$$\begin{array}{r|rrrrr} & 2 & 7 & -4 & -27 & -18 \\ & & & & & 0 \end{array}$$

$$\begin{array}{r|rrrrr} -1 & 2 & 7 & -4 & -27 & -18 \\ & & 0-2 & -5 & 9 & 18 \\ \hline & 2 & 5 & -9 & -18 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 2 & 5 & -9 & -18 \\ & & 0-6 & 3 & 18 \\ \hline & 2 & -1 & -6 & 0 \end{array}$$

$$\begin{aligned} & 2x^2 - x - 6 \\ & (2x+3)(x-2) = 0 \\ & x = -3/2 \text{ and } x = 2 \end{aligned}$$

Begin by setting the function equal to zero.

Once again, there are 18 possible zeros to the function. If Descarte's Rule of Signs is used, it may or may not help narrow down the choices for synthetic division.

This information was found in a previous example. Based on this, a chart may be constructed showing the possible combinations. Remember, this is a 4th degree polynomial, so each row must add up to 4.

+	-	i
1	3	0
1	1	2

$$\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}$$

-1 works as a zero of the function. There are now 3 zeros left. We can continue to test each zero, but we need to first rewrite the new polynomial.

$$2x^3 + 5x^2 - 9x - 18$$

The reason this must be done is to check using the rational zero test again. Using the rational zero test again could reduce the number of choices to work with, or the new polynomial may be factorable.

Here, we found that -3 works. The reason negative numbers are being used first here is because of the chart above. The chart says there is a greater chance of one of the negatives working rather than a positive, since there are potentially 3 negative zeros here and only one positive. Notice the new equation was used for the division.

This is now a factorable polynomial. Solve by factoring.

We now have all zeros of the polynomial function. They are $-3/2, -1, -3$ and 2

Be aware, the remaining polynomial may not be factorable. In that case, it will be necessary to either use the quadratic formula, or complete the square.