Exponential and Logarithmic Equations wksht 2

① $\log_x 12 = 3$

Exp Form

$x^3 = 12$

$\sqrt[3]{x^3} = \sqrt[3]{12}$

$x = \sqrt[3]{12}$

$x \approx 2.289$

② $3^{4x-5} = 5^{2x+1}$

$\log 3^{4x-5} = \log 5^{2x+1}$

$(4x-5)\log 3 = (2x+1)\log 5$

$4x\log 3 - 5\log 3 = 2x\log 5 + \log 5$

$4x\log 3 - 2x\log 5 = \log 5 + 5\log 3$

$x(4\log 3 - 2\log 5) = \log 5 + 5\log 3$

$x = \dfrac{\log 5 + 5\log 3}{4\log 3 - 2\log 5}$

$x \approx 6.042$

③ $4 + 3e^{2x+1} = 8$

$-4 \qquad -4$

$\dfrac{3e^{2x+1}}{3} = \dfrac{4}{3}$

$\ln e^{2x+1} = \ln \frac{4}{3}$

$(2x+1)\ \underline{\ln e = 1}$

$2x+1 = \ln \frac{4}{3}$

$-1 \qquad -1$

$2x = \ln \frac{4}{3} - 1$

$x = \dfrac{\ln\left(\frac{4}{3}\right) - 1}{2}$

$x = -0.356$

④ $\log_a 54$

$54 \to 2 \cdot 27 \to 3^3$

$\log_a (2 \cdot 3^3)$

$\log_a 2 + \log_a 3^3$

$\log_a 2 + 3\log_a 3$

$0.2544 + 3(0.5646)$

1.9482

⑤ $\log_a \frac{12}{5}$

$\log_a \left(\frac{2^2 \cdot 3}{5}\right)$

$\log_a 2^2 + \log_a 3 - \log_a 5$

$2\log_a 2 + \log_a 3 - \log_a 5$

$2(0.2544) + 0.5646 - 0.8271$

0.2463

⑥ $\log_a 160$

$160 \to 16 \cdot 10 \to 2^4 \cdot 2 \cdot 5$

$\log_a (2^5 \cdot 5)$

$\log_a 2^5 + \log_a 5$

$5\log_a 2 + \log_a 5$

$5(0.2544) + 0.8271$

2.0991

⑦ $\log_3 15$

$\frac{\log 15}{\log 3}$

2.465

⑧ $\log_7 56$

$\frac{\log 56}{\log 7}$

2.069

⑨ $\ln 5$

$\ln 5$

1.609

⑩ $\log_3(x+5) + \log_3(x+3) = \log_3 35$

$\log_3(x^2+8x+15) = \log_3 35$

$x^2 + 8x + 15 = 35$
$\quad -35 \quad -35$

$x^2 + 8x - 20 = 0$

$(x+10)(x-2) = 0$

$x+10=0 \quad x-2=0$

$\cancel{x=-10} \quad x=2$

$x=2$

$x=2$

$\log_3 7 + \log_3 5 = \log_3 35$ ✓

⑪ $2\log_3 x - \log_3(x-2) = 2$

$\log_3 \frac{x^2}{x-2} = 2$

Exp. Form

$3^2 = \frac{x^2}{x-2}$

$(x-2)\,9 = \frac{x^2}{x-2}\cancel{(x-2)}$

$9x - 18 = x^2$

$x^2 - 9x + 18 = 0$

$(x-6)(x-3) = 0$

$x-6=0 \quad x-3=0$

$x=6 \quad x=3$

$x=6$

$x=3$

$x=6$

$2\log_3 6 - \log_3 4 = 2$

$\log_3 36 - \log_3 4 = 2$

$\log_3 9 = 2$ ✓

$x=3$

$2\log_3 3 - \log_3 1 = 2$

$\log_3 9 - 0 = 2$ ✓

(12) $\log_2(x+3) + \log_2(x-3) = 4$

(13)

$\log_2(x^2-9) = 4$

Exp Form

$2^4 = x^2-9$

$16 = x^2 - 9$

$-16 \qquad -16$

$x^2 - 25 = 0$

$(x+5)(x-5)=0$

$\cancel{x=-5} \qquad x=5$

$x=5$

$\log_2 8 + \log_2 2 = 4$

$\log_2 16 = 4$ ✓

(14) $A = P\left(1+\frac{r}{n}\right)^{nt}$

$A=$

$P = 2500$

$r = .12$

$n = 4$

$t = 17$

$A = 2500\left(1+\frac{.12}{4}\right)^{(4)(17)}$

$A = 2500(1.03)^{68}$

$A = \$18,658.27$

(15) $A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = 30\,000$

$P=$

$r = .06$

$n = 12$

$t = 10$

$30\,000 = P\left(1+\frac{.06}{12}\right)^{(12)(10)}$

$\frac{30\,000}{(1.005)^{120}} = \frac{P(1.005)^{120}}{(1.005)^{120}}$

$P = \frac{30\,000}{(1.005)^{120}}$

$P = \$16.488.98$

(16) $A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = 15000$

$P = 2000$

$r = .085$

$n = 4$

$t =$

$$\frac{15000}{2000} = \frac{2000\left(1+\frac{.085}{4}\right)^{4t}}{2000}$$

$$\log\frac{15}{2} = \log\left(1+\frac{.085}{4}\right)^{4t}$$

$$\frac{\log 7.5}{4\log\left(1+\frac{.085}{4}\right)} = \frac{4t\log\left(1+\frac{.085}{4}\right)}{4\log\left(1+\frac{.085}{4}\right)}$$

$$t = \frac{\log 7.5}{4\log\left(1+\frac{.085}{4}\right)}$$

$t = 23.95$ round up

$t = 24$ years

(17) $P = 400 - .06\,e^{0.003x}$ ← solve for x

$$99 = 400 - .06\,e^{0.003x}$$

$-400 \quad -400$

$$\frac{-301}{-0.06} = \frac{-0.06\,e^{0.003x}}{-0.06}$$

Decimal too big

$$\ln\frac{301}{.06} = \ln e^{0.003x}$$

$$\ln\frac{301}{.06} = 0.003x\,\ln e$$

$\ln e = 1$

$$\frac{0.003x}{0.003} = \frac{\ln\left(\frac{301}{.06}\right)}{0.003}$$

$$x = \frac{\ln\left(\frac{301}{.06}\right)}{0.003}$$

$x \approx 2840.2$

(18) $P = 30e^{kt}$

1990

$$52 = 30e^{k(-10)}$$

$$\frac{52}{30} = \frac{30}{30}e^{-10k}$$

$$\ln\frac{52}{30} = \ln e^{-10k}$$

$$\ln\frac{52}{30} = -10k\,\ln \cancel{e}$$

$\ln e = 1$

$$\frac{-10k}{-10} = \frac{\ln\frac{52}{30}}{-10} \quad k \approx -0.055$$

2012

$$P = 30e^{(-0.055)(12)}$$

$P \approx 15.506$

in thousands

In 2012 population will be approximately 15,506

(19) $R = \log_{10} \frac{I}{I_0}$

$I_0 = 1$

(A) $6.5 = \log_{10} \frac{I}{1}$

$\log_{10} I = 6.5$

Exp. Form

$10^{6.5} = I$

$I = 3{,}162{,}277.66$

(B) $3.2 = \log_{10} \frac{I}{1}$

$\log_{10} I = 3.2$

Ex Form

$10^{3.2} = I$

$I = 1{,}584.89$

(C) $R = \log_{10} \frac{325\text{ own}}{1}$

$R = \log_{10} 325\text{ own}$

$R = 5.5$ on Richter Scale