FINDING THE INVERSE OF A MATRIX

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$a_{23} \rightarrow$ the element in row 2 column 3.

$$A \quad \cdot \quad A^{-1} \quad = \quad I$$

$$\begin{bmatrix} \text{Matrix A} \end{bmatrix} \begin{bmatrix} \text{Inverse of} \\ \text{Matrix A} \end{bmatrix} = \begin{bmatrix} \text{Identity} \\ \text{Matrix} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A Non-Square Matrix **DOES NOT** have an Inverse.

Show that matrix B is the inverse of matrix A.

$$A = \begin{bmatrix} -2 & 0 & 1 & 0 \\ 1 & -1 & -3 & 0 \\ -2 & -1 & 0 & -2 \\ 0 & 1 & 3 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} -3 & -3 & 1 & -2 \\ 12 & 14 & -5 & 10 \\ -5 & -6 & 2 & -4 \\ -3 & -4 & 1 & -3 \end{bmatrix}$$

FINDING THE INVERSE OF A 2X2 MATRIX

There are two ways to find the inverse of a 2x2 matrix. The first is using an augmented matrix

Given the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, set up an augmented matrix like this: $\left[\begin{array}{cc|cc} a & b & 1 & 0 \\ c & d & 0 & 1 \end{array}\right]$

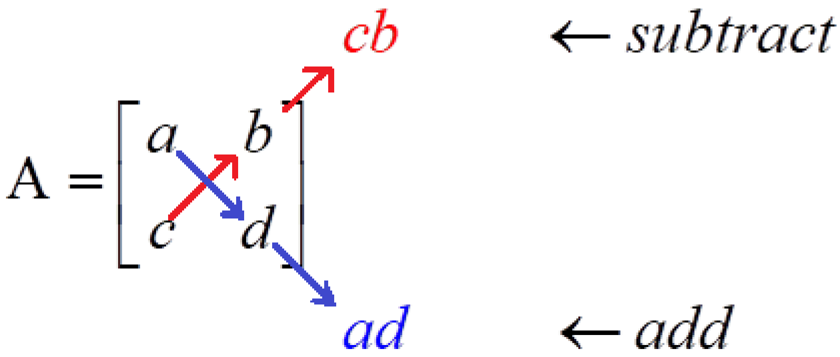
You can see on the right side of the matrix is the identity matrix for a 2x2. The goal is to make the left side look like the right using elementary row operations. Once the left side is the identity matrix, what is left on the right side is the inverse of the 2x2.

Example

Given matrix $A = \begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$, find A^{-1}.

$$\left[\begin{array}{cc|cc} 3 & -1 & 1 & 0 \\ -2 & 2 & 0 & 1 \end{array}\right]$$

The second way to find the inverse of a 2x2 matrix is by using a formula.



$$A^{-1} = \frac{1}{ad - cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

become opposites

switch places

Example

Given matrix $A = \begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$, find A^{-1}.

$$\begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$$

There is only one way to find the inverse of a 3x3 or any larger square matrix. That is using the following method.

$$\text{Given matrix A} = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix}$$

$$\text{Set up} \left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 3 & 7 & 9 & 0 & 1 & 0 \\ -1 & -4 & -7 & 0 & 0 & 1 \end{array}\right]$$

USING INVERSE MATRICES TO SOLVE SYSTEMS OF EQUATIONS

If given the system

$$\begin{cases} x + y + z = -1 \\ 3x + 5y + 4z = 2 \\ 3x + 6y + 5z = 0 \end{cases}$$

Set up the following:

$$\underset{3\text{x}3}{A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 3 & 6 & 5 \end{bmatrix}} \quad \text{and} \quad \underset{3\text{x}1}{B = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}}$$

Coefficient Matrix Constants

The solution to the system is given by:

$$A^{-1} \cdot B$$

If the calculator says it is a singular matrix, that means the matrix is not invertible.

(No inverse can be found for matrix A)

This means the system either has **No Solution** or **Infinite Solutions**.

If the system has no solution

There is nothing you can do, you are done with the problem

If the system has inifinite solutions

The answers are expressions.

Solve the following Systems of equations.

1. $\begin{cases} 3x + 2y = 10 \\ 2x + 5y = 3 \end{cases}$

2. $\begin{cases} 2x + 4y + z = 1 \\ x - 2y - 3z = 2 \\ x + y - z = -1 \end{cases}$

3. $\begin{cases} 2x + 3y + 5z = 4 \\ 3x + 5y + 9z = 7 \\ 5x + 9y + 17z = 13 \end{cases}$

4. $\begin{cases} 2x + y - z = 7 \\ x - 2y + 2z = -9 \\ 3x - y + z = 5 \end{cases}$