Think Like a Scientist

Scientists have a particular way of looking at the world, or scientific habits of mind. Whenever you ask a question and explore possible answers, you use many of the same skills that scientists do. Some of these skills are described on this page.

Observing

When you use one or more of your five senses to gather information about the world, you are **observing.** Hearing a dog bark, counting twelve green seeds, and smelling smoke are all observations. To increase the power of their senses, scientists sometimes use microscopes, telescopes, or other instruments that help them make more detailed observations.

An observation must be an accurate report of what your senses detect. It is important to keep careful records of your observations in science class by writing or drawing in a notebook. The information collected through observations is called evidence, or data.

Inferring

When you interpret an observation, you are **inferring,** or making an inference. For example, if you hear your dog barking, you may infer that someone is at your front door. To make this inference, you combine the evidence—the barking dog—and your experience or knowledge—you know that your dog barks when strangers approach—to reach a logical conclusion.

Notice that an inference is not a fact; it is only one of many possible interpretations for an observation. For example, your dog may be barking because it wants to go for a walk. An inference may turn out to be incorrect even if it is based on accurate observations and logical reasoning. The only way to find out if an inference is correct is to investigate further.

Predicting

When you listen to the weather forecast, you hear many predictions about the next day's weather—what the temperature will be, whether it will rain, and how windy it will be. Weather forecasters use observations and knowledge of weather patterns to predict the weather. The skill of **predicting** involves making an inference about a future event based on current evidence or past experience.

Because a prediction is an inference, it may prove to be false. In science class, you can test some of your predictions by doing experiments. For example, suppose you predict that larger paper airplanes can fly farther than smaller airplanes. How could you test your prediction?

Activity

Use the photograph to answer the questions below.

Observing Look closely at the photograph. List at least three observations.

Inferring Use your observations to make an inference about what has happened. What experience or knowledge did you use to make the inference?

Predicting Predict what will happen next. On what evidence or experience do you base your prediction?

Classifying

Could you imagine searching for a book in the library if the books were shelved in no particular order? Your trip to the library would be an all-day event! Luckily, librarians group together books on similar topics or by the same author. Grouping together items that are alike in some way is called **classifying.** You can classify items in many ways: by size, by shape, by use, and by other important characteristics.

Like librarians, scientists use the skill of classifying to organize information and objects. When things are sorted into groups, the relationships among them become easier to understand.

Activity

Classify the objects in the photograph into two groups based on any characteristic you choose. Then use another characteristic to classify the objects into three groups.

Making Models

Have you ever drawn a picture to help someone understand what you were saying? Such a drawing is one type of model. A model is a picture, diagram, computer image, or other representation of a complex object or process. **Making models** helps people understand things that they cannot observe directly.

Scientists often use models to represent things that are either very large or very small, such as the planets in the solar system, or the parts of a cell. Such models are physical models—drawings or three-dimensional structures that look like the real thing. Other models are mental models—mathematical equations or words that describe how something works.

Activity

This student is using a model to demonstrate what causes day and night on Earth. What do the flashlight and the tennis ball in the model represent?

Communicating

Whenever you talk on the phone, write a report, or listen to your teacher at school, you are communicating. **Communicating** is the process of sharing ideas and information with other people. Communicating effectively requires many skills, including writing, reading, speaking, listening, and making models.

Scientists communicate to share results, information, and opinions. Scientists often communicate about their work in journals, over the telephone, in letters, and on the Internet. They also attend scientific meetings where they share their ideas with one another in person.

Activity

On a sheet of paper, write out clear, detailed directions for tying your shoe. Then exchange directions with a partner. Follow your partner's directions exactly. How successful were you at tying your shoe? How could your partner have communicated more clearly?

Making Measurements

By measuring, scientists can express their observations more precisely and communicate more information about what they observe.

Measuring in SI

The standard system of measurement used by scientists around the world is known as the International System of Units, which is abbreviated as SI (**Système International d'Unités,** in French). SI units are easy to use because they are based on multiples of 10. Each unit is ten times larger than the next smallest unit and one tenth the size of the next largest unit. The table lists the prefixes used to name the most common SI units.

Common SI Prefixes

Prefix	Symbol	Meaning
kilo-	k	1,000
hecto-	h	100
deka-	da	10
deci-	d	0.1 (one tenth)
centi-	c	0.01 (one hundredth)
milli-	m	0.001 (one thousandth)

Length To measure length, or the distance between two points, the unit of measure is the **meter (m).** The distance from the floor to a doorknob is approximately one meter. Long distances, such as the distance between two cities, are measured in kilometers (km). Small lengths are measured in centimeters (cm) or millimeters (mm). Scientists use metric rulers and meter sticks to measure length.

Common Conversions

1 km	=	1,000 m
1 m	=	100 cm
1 m	=	1,000 mm
1 cm	=	10 mm

Activity

The larger lines on the metric ruler in the picture show centimeter divisions, while the smaller, unnumbered lines show millimeter divisions. How many centimeters long is the shell? How many millimeters long is it?

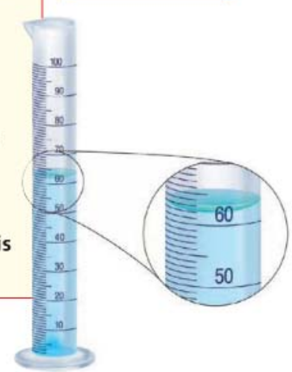
Liquid Volume To measure the volume of a liquid, or the amount of space it takes up, you will use a unit of measure known as the **liter (L).** One liter is the approximate volume of a medium-size carton of milk. Smaller volumes are measured in milliliters (mL). Scientists use graduated cylinders to measure liquid volume.

Activity

The graduated cylinder in the picture is marked in milliliter divisions. Notice that the water in the cylinder has a curved surface. This curved surface is called the *meniscus*. To measure the volume, you must read the level at the lowest point of the meniscus. What is the volume of water in this graduated cylinder?

Common Conversion

1 L	=	1,000 mL



Mass To measure mass, or the amount of matter in an object, you will use a unit of measure known as the **gram (g).** One gram is approximately the mass of a paper clip. Larger masses are measured in kilograms (kg). Scientists use a balance to find the mass of an object.

Common Conversion
1 kg = 1,000 g

Activity

The mass of the potato in the picture is measured in kilograms. What is the mass of the potato? Suppose a recipe for potato salad called for one kilogram of potatoes. About how many potatoes would you need?

Temperature To measure the temperature of a substance, you will use the **Celsius scale.** Temperature is measured in degrees Celsius (°C) using a Celsius thermometer. Water freezes at 0°C and boils at 100°C.

Time The unit scientists use to measure time is the **second (s).**

Activity

What is the temperature of the liquid in degrees Celsius?

Converting SI Units

To use the SI system, you must know how to convert between units. Converting from one unit to another involves the skill of **calculating,** or using mathematical operations. Converting between SI units is similar to converting between dollars and dimes because both systems are based on multiples of ten.

Suppose you want to convert a length of 80 centimeters to meters. Follow these steps to convert between units.

1. Begin by writing down the measurement you want to convert—in this example, 80 centimeters.
2. Write a conversion factor that represents the relationship between the two units you are converting. In this example, the relationship is 1 meter = 100 centimeters. Write this conversion factor as a fraction, making sure to place the units you are converting from (centimeters, in this example) in the denominator.
3. Multiply the measurement you want to convert by the fraction. When you do this, the units in the first measurement will cancel out with the units in the denominator. Your answer will be in the units you are converting to (meters, in this example).

Example

80 centimeters = ■ meters

$$80 \text{ centimeters} \times \frac{1 \text{ meter}}{100 \text{ centimeters}} = \frac{80 \text{ meters}}{100}$$

$$= 0.8 \text{ meters}$$

Activity

Convert between the following units.

1. 600 millimeters = ■ meters
2. 0.35 liters = ■ milliliters
3. 1,050 grams = ■ kilograms

Conducting a Scientific Investigation

In some ways, scientists are like detectives, piecing together clues to learn about a process or event. One way that scientists gather clues is by carrying out experiments. An experiment tests an idea in a careful, orderly manner. Although experiments do not all follow the same steps in the same order, many follow a pattern similar to the one described here.

Posing Questions

Experiments begin by asking a scientific question. A scientific question is one that can be answered by gathering evidence. For example, the question "Which freezes faster—fresh water or salt water?" is a scientific question because you can carry out an investigation and gather information to answer the question.

Developing a Hypothesis

The next step is to form a hypothesis. A **hypothesis** is a possible explanation for a set of observations or answer to a scientific question. A hypothesis may incorporate observations, concepts, principles, and theories about the natural world. Hypotheses lead to predictions that can be tested. A prediction can be worded as an *If . . . then . . .* statement. For example, a prediction might be *"If I add salt to fresh water, then the water will take longer to freeze."* A prediction worded this way serves as a rough outline of the experiment you should perform.

Designing an Experiment

Next you need to plan a way to test your hypothesis. Your plan should be written out as a step-by-step procedure and should describe the observations or measurements you will make.

Two important steps involved in designing an experiment are controlling variables and forming operational definitions.

Controlling Variables In a well-designed experiment, you need to keep all variables the same except for one. A **variable** is any factor that can change in an experiment. The factor that you change is called the **manipulated variable**. In this experiment, the manipulated variable is the amount of salt added to the water. Other factors, such as the amount of water or the starting temperature, are kept constant.

The factor that changes as a result of the manipulated variable is called the **responding variable.** The responding variable is what you measure or observe to obtain your results. In this experiment, the responding variable is how long the water takes to freeze.

An experiment in which all factors except one are kept constant is called a **controlled experiment.** Most controlled experiments include a test called the control. In this experiment, Container 3 is the control. Because no salt is added to Container 3, you can compare the results from the other containers to it. Any difference in results must be due to the addition of salt alone.

Forming Operational Definitions Another important aspect of a well-designed experiment is having clear operational definitions. An **operational definition** is a statement that describes how a particular variable is to be measured or how a term is to be defined. For example, in this experiment, how will you determine if the water has frozen? You might decide to insert a stick in each container at the start of the experiment. Your operational definition of "frozen" would be the time at which the stick can no longer move.

Experimental Procedure

1. Fill 3 containers with 300 milliliters of cold tap water.
2. Add 10 grams of salt to Container 1; stir. Add 20 grams of salt to Container 2; stir. Add no salt to Container 3.
3. Place the 3 containers in a freezer.
4. Check the containers every 15 minutes. Record your observations.

Interpreting Data

The observations and measurements you make in an experiment are called **data.** At the end of an experiment, you need to analyze the data to look for any patterns or trends. Patterns often become clear if you organize your data in a data table or graph. Then think through what the data reveal. Do they support your hypothesis? Do they point out a flaw in your experiment? Do you need to collect more data?

Drawing Conclusions

A **conclusion** is a statement that sums up what you have learned from an experiment. When you draw a conclusion, you need to decide whether the data you collected support your hypothesis or not. You may need to repeat an experiment several times before you can draw any conclusions from it. Conclusions often lead you to pose new questions and plan new experiments to answer them.

Activity

Is a ball's bounce affected by the height from which it is dropped? Using the steps just described, plan a controlled experiment to investigate this problem.

Technology Design Skills

Engineers are people who use scientific and technological knowledge to solve practical problems. To design new products, engineers usually follow the process described here, even though they may not follow these steps in the exact order. As you read the steps, think about how you might apply them in technology labs.

Identify a Need

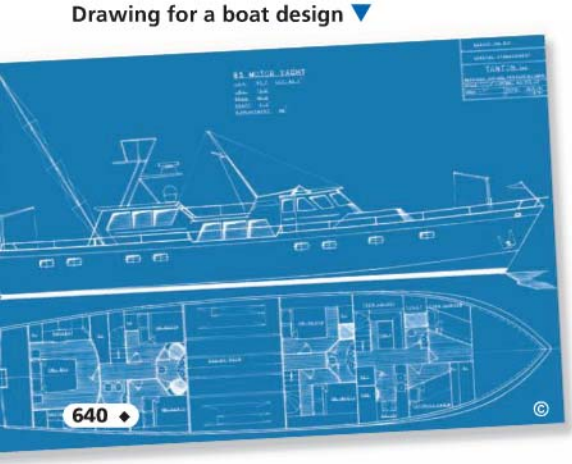
Before engineers begin designing a new product, they must first identify the need they are trying to meet. For example, suppose you are a member of a design team in a company that makes toys. Your team has identified a need: a toy boat that is inexpensive and easy to assemble.

Research the Problem

Engineers often begin by gathering information that will help them with their new design. This research may include finding articles in books, magazines, or on the Internet. It may also include talking to other engineers who have solved similar problems. Engineers often perform experiments related to the product they want to design.

For your toy boat, you could look at toys that are similar to the one you want to design. You might do research on the Internet. You could also test some materials to see whether they will work well in a toy boat.

Drawing for a boat design ▼



Design a Solution

Research gives engineers information that helps them design a product. When engineers design new products, they usually work in teams.

Generating Ideas Often design teams hold brainstorming meetings in which any team member can contribute ideas. **Brainstorming** is a creative process in which one team member's suggestions often spark ideas in other group members. Brainstorming can lead to new approaches to solving a design problem.

Evaluating Constraints During brainstorming, a design team will often come up with several possible designs. The team must then evaluate each one.

As part of their evaluation, engineers consider constraints. **Constraints** are factors that limit or restrict a product design. Physical characteristics, such as the properties of materials used to make your toy boat, are constraints. Money and time are also constraints. If the materials in a product cost a lot, or if the product takes a long time to make, the design may be impractical.

Making Trade-offs Design teams usually need to make trade-offs. In a **trade-off,** engineers give up one benefit of a proposed design in order to obtain another. In designing your toy boat, you will have to make trade-offs. For example, suppose one material is sturdy but not fully waterproof. Another material is more waterproof, but breakable. You may decide to give up the benefit of sturdiness in order to obtain the benefit of waterproofing.

Build and Evaluate a Prototype

Once the team has chosen a design plan, the engineers build a prototype of the product. A **prototype** is a working model used to test a design. Engineers evaluate the prototype to see whether it works well, is easy to operate, is safe to use, and holds up to repeated use.

Think of your toy boat. What would the prototype be like? Of what materials would it be made? How would you test it?

Troubleshoot and Redesign

Few prototypes work perfectly, which is why they need to be tested. Once a design team has tested a prototype, the members analyze the results and identify any problems. The team then tries to **troubleshoot,** or fix the design problems. For example, if your toy boat leaks or wobbles, the boat should be redesigned to eliminate those problems.

Communicate the Solution

A team needs to communicate the final design to the people who will manufacture and use the product. To do this, teams may use sketches, detailed drawings, computer simulations, and word descriptions.

Activity

You can use the technology design process to design and build a toy boat.

Research and Investigate

1. Visit the library or go online to research toy boats.
2. Investigate how a toy boat can be powered, including wind, rubber bands, or baking soda and vinegar.
3. Brainstorm materials, shapes, and steering for your boat.

Design and Build

4. Based on your research, design a toy boat that
   - is made of readily available materials
   - is no larger than 15 cm long and 10 cm wide
   - includes a power system, a rudder, and an area for cargo
   - travels 2 meters in a straight line carrying a load of 20 pennies
5. Sketch your design and write a step-by-step plan for building your boat. After your teacher approves your plan, build your boat.

Evaluate and Redesign

6. Test your boat, evaluate the results, and troubleshoot any problems.
7. Based on your evaluation, redesign your toy boat so it performs better.

Creating Data Tables and Graphs

How can you make sense of the data in a science experiment? The first step is to organize the data to help you understand them. Data tables and graphs are helpful tools for organizing data.

Data Tables

You have gathered your materials and set up your experiment. But before you start, you need to plan a way to record what happens during the experiment. By creating a data table, you can record your observations and measurements in an orderly way.

Suppose, for example, that a scientist conducted an experiment to find out how many Calories people of different body masses burn while doing various activities. The data table shows the results.

Notice in this data table that the manipulated variable (body mass) is the heading of one column. The responding variable (for Experiment 1, the number of Calories burned while bicycling) is the heading of the next column. Additional columns were added for related experiments.

Calories Burned in 30 Minutes

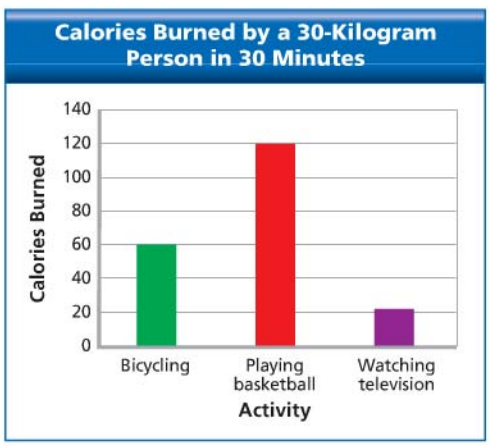
Body Mass	Experiment 1: Bicycling	Experiment 2: Playing Basketball	Experiment 3: Watching Television
30 kg	60 Calories	120 Calories	21 Calories
40 kg	77 Calories	164 Calories	27 Calories
50 kg	95 Calories	206 Calories	33 Calories
60 kg	114 Calories	248 Calories	38 Calories

Bar Graphs

To compare how many Calories a person burns doing various activities, you could create a bar graph. A bar graph is used to display data in a number of separate, or distinct, categories. In this example, bicycling, playing basketball, and watching television are the three categories.

To create a bar graph, follow these steps.

1. On graph paper, draw a horizontal, or x-, axis and a vertical, or y-, axis.
2. Write the names of the categories to be graphed along the horizontal axis. Include an overall label for the axis as well.
3. Label the vertical axis with the name of the responding variable. Include units of measurement. Then create a scale along the axis by marking off equally spaced numbers that cover the range of the data collected.
4. For each category, draw a solid bar using the scale on the vertical axis to determine the height. Make all the bars the same width.
5. Add a title that describes the graph.

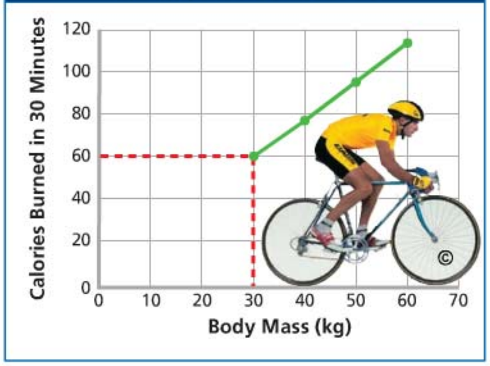


Line Graphs

To see whether a relationship exists between body mass and the number of Calories burned while bicycling, you could create a line graph. A line graph is used to display data that show how one variable (the responding variable) changes in response to another variable (the manipulated variable). You can use a line graph when your manipulated variable is **continuous,** that is, when there are other points between the ones that you tested. In this example, body mass is a continuous variable because there are other body masses between 30 and 40 kilograms (for example, 31 kilograms). Time is another example of a continuous variable.

Line graphs are powerful tools because they allow you to estimate values for conditions that you did not test in the experiment. For example, you can use the line graph to estimate that a 35-kilogram person would burn 68 Calories while bicycling.

Effect of Body Mass on Calories Burned While Bicycling



To create a line graph, follow these steps.

1. On graph paper, draw a horizontal, or *x*-, axis and a vertical, or *y*-, axis.
2. Label the horizontal axis with the name of the manipulated variable. Label the vertical axis with the name of the responding variable. Include units of measurement.
3. Create a scale on each axis by marking off equally spaced numbers that cover the range of the data collected.
4. Plot a point on the graph for each piece of data. In the line graph above, the dotted lines show how to plot the first data point (30 kilograms and 60 Calories). Follow an imaginary vertical line extending up from the horizontal axis at the 30-kilogram mark. Then follow an imaginary horizontal line extending across from the vertical axis at the 60-Calorie mark. Plot the point where the two lines intersect.
5. Connect the plotted points with a solid line. (In some cases, it may be more appropriate to draw a line that shows the general trend of the plotted points. In those cases, some of the points may fall above or below the line. Also, not all graphs are linear. It may be more appropriate to draw a curve to connect the points.)
6. Add a title that identifies the variables or relationship in the graph.

Activity

Create line graphs to display the data from Experiment 2 and Experiment 3 in the data table.

Activity

You read in the newspaper that a total of 4 centimeters of rain fell in your area in June, 2.5 centimeters fell in July, and 1.5 centimeters fell in August. What type of graph would you use to display these data? Use graph paper to create the graph.

Circle Graphs

Like bar graphs, circle graphs can be used to display data in a number of separate categories. Unlike bar graphs, however, circle graphs can only be used when you have data for *all* the categories that make up a given topic. A circle graph is sometimes called a pie chart. The pie represents the entire topic, while the slices represent the individual categories. The size of a slice indicates what percentage of the whole a particular category makes up.

The data table below shows the results of a survey in which 24 teenagers were asked to identify their favorite sport. The data were then used to create the circle graph at the right.

Favorite Sports

Sport	Students
Soccer	8
Basketball	6
Bicycling	6
Swimming	4

To create a circle graph, follow these steps.

1. Use a compass to draw a circle. Mark the center with a point. Then draw a line from the center point to the top of the circle.
2. Determine the size of each "slice" by setting up a proportion where x equals the number of degrees in a slice. (*Note:* A circle contains 360 degrees.) For example, to find the number of degrees in the "soccer" slice, set up the following proportion:

$$\frac{\text{Students who prefer soccer}}{\text{Total number of students}} = \frac{x}{\text{Total number of degrees in a circle}}$$

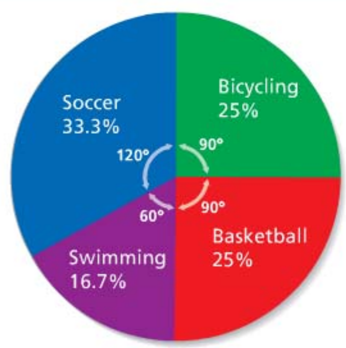
$$\frac{8}{24} = \frac{x}{360}$$

Cross-multiply and solve for x.

$$24x = 8 \times 360$$
$$x = 120$$

The "soccer" slice should contain 120 degrees.

Sports That Teens Prefer



3. Use a protractor to measure the angle of the first slice, using the line you drew to the top of the circle as the 0° line. Draw a line from the center of the circle to the edge for the angle you measured.
4. Continue around the circle by measuring the size of each slice with the protractor. Start measuring from the edge of the previous slice so the wedges do not overlap. When you are done, the entire circle should be filled in.
5. Determine the percentage of the whole circle that each slice represents. To do this, divide the number of degrees in a slice by the total number of degrees in a circle (360), and multiply by 100%. For the "soccer" slice, you can find the percentage as follows:

$$\frac{120}{360} \times 100\% = 33.3\%$$

6. Use a different color for each slice. Label each slice with the category and with the percentage of the whole it represents.
7. Add a title to the circle graph.

Activity

In a class of 28 students, 12 students take the bus to school, 10 students walk, and 6 students ride their bicycles. Create a circle graph to display these data.

Math Review

Scientists use math to organize, analyze, and present data. This appendix will help you review some basic math skills.

Mean, Median, and Mode

The **mean** is the average, or the sum of the data divided by the number of data items. The middle number in a set of ordered data is called the **median.** The **mode** is the number that appears most often in a set of data.

Example

A scientist counted the number of distinct songs sung by seven different male birds and collected the data shown below.

Male Bird Songs

Bird	A	B	C	D	E	F	G
Number of Songs	36	29	40	35	28	36	27

To determine the mean number of songs, add the total number of songs and divide by the number of data items—in this case, the number of male birds.

$$\text{Mean} = \frac{231}{7} = 33 \text{ songs}$$

To find the median number of songs, arrange the data in numerical order and find the number in the middle of the series.

27 28 29 35 36 36 40

The number in the middle is 35, so the median number of songs is 35.

The mode is the value that appears most frequently. In the data, 36 appears twice, while each other item appears only once. Therefore, 36 songs is the mode.

Practice

Find out how many minutes it takes each student in your class to get to school. Then find the mean, median, and mode for the data.

Probability

Probability is the chance that an event will occur. Probability can be expressed as a ratio, a fraction, or a percentage. For example, when you flip a coin, the probability that the coin will land heads up is 1 in 2, or $\frac{1}{2}$, or 50 percent.

The probability that an event will happen can be expressed in the following formula.

$$P(\text{event}) = \frac{\text{Number of times the event can occur}}{\text{Total number of possible events}}$$

Example

A paper bag contains 25 blue marbles, 5 green marbles, 5 orange marbles, and 15 yellow marbles. If you close your eyes and pick a marble from the bag, what is the probability that it will be yellow?

$$P(\text{yellow marbles}) = \frac{\text{15 yellow marbles}}{\text{50 marbles total}}$$

$$P = \frac{15}{50}, \text{ or } \frac{3}{10}, \text{ or } 30\%$$

Practice

Each side of a cube has a letter on it. Two sides have *A*, three sides have *B*, and one side has *C*. If you roll the cube, what is the probability that *A* will land on top?

Area

The **area** of a surface is the number of square units that cover it. The front cover of your textbook has an area of about 600 cm^2.

Area of a Rectangle and a Square
To find the area of a rectangle, multiply its length times its width. The formula for the area of a rectangle is

$$A = \ell \times w, \text{ or } A = \ell w$$

Since all four sides of a square have the same length, the area of a square is the length of one side multiplied by itself, or squared.

$$A = s \times s, \text{ or } A = s^2$$

Example

A scientist is studying the plants in a field that measures 75 m × 45 m. What is the area of the field?

$$A = \ell \times w$$
$$A = 75 \text{ m} \times 45 \text{ m}$$
$$A = 3{,}375 \text{ m}^2$$

Area of a Circle
The formula for the area of a circle is

$$A = \pi \times r \times r, \text{ or } A = \pi r^2$$

The length of the radius is represented by r, and the value of π is approximately $\frac{22}{7}$.

Example

Find the area of a circle with a radius of 14 cm.

$$A = \pi r^2$$
$$A = \frac{22}{7} \times 14 \text{ cm} \times 14 \text{ cm}$$
$$A = 616 \text{ cm}^2$$

Practice

Find the area of a circle that has a radius of 21 m.

Circumference

The distance around a circle is called the circumference. The formula for finding the circumference of a circle is

$$C = 2 \times \pi \times r, \text{ or } C = 2\pi r$$

Example

The radius of a circle is 35 cm. What is its circumference?

$$C = 2\pi r$$
$$C = 2 \times \frac{22}{7} \times 35 \text{ cm}$$
$$C = 220 \text{ cm}$$

Practice

What is the circumference of a circle with a radius of 28 m?

Volume

The volume of an object is the number of cubic units it contains. The volume of a wastebasket, for example, might be about 26,000 cm^3.

Volume of a Rectangular Object
To find the volume of a rectangular object, multiply the object's length times its width times its height.

$$V = \ell \times w \times h, \text{ or } V = \ell wh$$

Example

Find the volume of a box with length 24 cm, width 12 cm, and height 9 cm.

$$V = \ell wh$$
$$V = 24 \text{ cm} \times 12 \text{ cm} \times 9 \text{ cm}$$
$$V = 2{,}592 \text{ cm}^3$$

Practice

What is the volume of a rectangular object with length 17 cm, width 11 cm, and height 6 cm?

Fractions

A **fraction** is a way to express a part of a whole. In the fraction $\frac{4}{7}$, 4 is the numerator and 7 is the denominator.

Adding and Subtracting Fractions To add or subtract two or more fractions that have a common denominator, first add or subtract the numerators. Then write the sum or difference over the common denominator.

To find the sum or difference of fractions with different denominators, first find the least common multiple of the denominators. This is known as the least common denominator. Then convert each fraction to equivalent fractions with the least common denominator. Add or subtract the numerators. Then write the sum or difference over the common denominator.

Example

$$\frac{5}{6} - \frac{3}{4} = \frac{10}{12} - \frac{9}{12} = \frac{10 - 9}{12} = \frac{1}{12}$$

Multiplying Fractions To multiply two fractions, first multiply the two numerators, then multiply the two denominators.

Example

$$\frac{5}{6} \times \frac{2}{3} = \frac{5 \times 2}{6 \times 3} = \frac{10}{18} = \frac{5}{9}$$

Dividing Fractions Dividing by a fraction is the same as multiplying by its reciprocal. Reciprocals are numbers whose numerators and denominators have been switched. To divide one fraction by another, first invert the fraction you are dividing by—in other words, turn it upside down. Then multiply the two fractions.

Example

$$\frac{2}{5} \div \frac{7}{8} = \frac{2}{5} \times \frac{8}{7} = \frac{2 \times 8}{5 \times 7} = \frac{16}{35}$$

Practice

Solve the following: $\frac{3}{7} \div \frac{4}{5}$.

Decimals

Fractions whose denominators are 10, 100, or some other power of 10 are often expressed as decimals. For example, the fraction $\frac{9}{10}$ can be expressed as the decimal 0.9, and the fraction $\frac{7}{100}$ can be written as 0.07.

Adding and Subtracting With Decimals To add or subtract decimals, line up the decimal points before you carry out the operation.

Example

$$\begin{array}{r} 27.4 \\ +\ 6.19 \\ \hline 33.59 \end{array} \qquad \begin{array}{r} 278.635 \\ -\ 191.4 \\ \hline 87.235 \end{array}$$

Multiplying With Decimals When you multiply two numbers with decimals, the number of decimal places in the product is equal to the total number of decimal places in each number being multiplied.

Example

46.2 (one decimal place)
× 2.37 (two decimal places)
109.494 (three decimal places)

Dividing With Decimals To divide a decimal by a whole number, put the decimal point in the quotient above the decimal point in the dividend.

Example

$$15.5 \div 5$$

$$5\overline{)15.5} = 3.1$$

To divide a decimal by a decimal, you need to rewrite the divisor as a whole number. Do this by multiplying both the divisor and dividend by the same multiple of 10.

Example

$$1.68 \div 4.2 = 1.68 \div 4.2$$

$$42\overline{)16.8} = 0.4$$

Practice

Multiply 6.21 by 8.5.

Ratio and Proportion

A **ratio** compares two numbers by division. For example, suppose a scientist counts 800 wolves and 1,200 moose on an island. The ratio of wolves to moose can be written as a fraction, $\frac{800}{1,200}$, which can be reduced to $\frac{2}{3}$. The same ratio can also be expressed as 2 to 3 or 2 : 3.

A **proportion** is a mathematical sentence saying that two ratios are equivalent. For example, a proportion could state that $\frac{800 \text{ wolves}}{1,200 \text{ moose}} = \frac{2 \text{ wolves}}{3 \text{ moose}}$. You can sometimes set up a proportion to determine or estimate an unknown quantity. For example, suppose a scientist counts 25 beetles in an area of 10 square meters. The scientist wants to estimate the number of beetles in 100 square meters.

Example

1. Express the relationship between beetles and area as a ratio: $\frac{25}{10}$, simplified to $\frac{5}{2}$.
2. Set up a proportion, with x representing the number of beetles. The proportion can be stated as $\frac{5}{2} = \frac{x}{100}$.
3. Begin by cross-multiplying. In other words, multiply each fraction's numerator by the other fraction's denominator.

$$5 \times 100 = 2 \times x, \text{ or } 500 = 2x$$

4. To find the value of x, divide both sides by 2. The result is 250, or 250 beetles in 100 square meters.

Practice

Find the value of x in the following proportion: $\frac{6}{7} = \frac{x}{49}$.

Percentage

A **percentage** is a ratio that compares a number to 100. For example, there are 37 granite rocks in a collection that consists of 100 rocks. The ratio $\frac{37}{100}$ can be written as 37%. Granite rocks make up 37% of the rock collection.

You can calculate percentages of numbers other than 100 by setting up a proportion.

Example

Rain falls on 9 days out of 30 in June. What percentage of the days in June were rainy?

$$\frac{9 \text{ days}}{30 \text{ days}} = \frac{d\%}{100\%}$$

To find the value of d, begin by cross-multiplying, as for any proportion:

$$9 \times 100 = 30 \times d \qquad d = \frac{900}{30} \qquad d = 30$$

Practice

There are 300 marbles in a jar, and 42 of those marbles are blue. What percentage of the marbles are blue?

Significant Figures

The **precision** of a measurement depends on the instrument you use to take the measurement. For example, if the smallest unit on the ruler is millimeters, then the most precise measurement you can make will be in millimeters.

The sum or difference of measurements can only be as precise as the least precise measurement being added or subtracted. Round your answer so that it has the same number of digits after the decimal as the least precise measurement. Round up if the last digit is 5 or more, and round down if the last digit is 4 or less.

Example

Subtract a temperature of 5.2°C from the temperature 75.46°C.

$$75.46 - 5.2 = 70.26$$

5.2 has the fewest digits after the decimal, so it is the least precise measurement. Since the last digit of the answer is 6, round up to 3. The most precise difference between the measurements is 70.3°C.

Practice

Add 26.4 m to 8.37 m. Round your answer according to the precision of the measurements.

Significant figures are the number of nonzero digits in a measurement. Zeros between nonzero digits are also significant. For example, the measurements 12,500 L, 0.125 cm, and 2.05 kg all have three significant figures. When you multiply and divide measurements, the one with the fewest significant figures determines the number of significant figures in your answer.

Example

Multiply 110 g by 5.75 g.

$$110 \times 5.75 = 632.5$$

Because 110 has only two significant figures, round the answer to 630 g.

Scientific Notation

A **factor** is a number that divides into another number with no remainder. In the example, the number 3 is used as a factor four times.

An **exponent** tells how many times a number is used as a factor. For example, $3 \times 3 \times 3 \times 3$ can be written as 3^4. The exponent "4" indicates that the number 3 is used as a factor four times. Another way of expressing this is to say that 81 is equal to 3 to the fourth power.

Example

$$3^4 = 3 \times 3 \times 3 \times 3 = 81$$

Scientific notation uses exponents and powers of ten to write very large or very small numbers in shorter form. When you write a number in scientific notation, you write the number as two factors. The first factor is any number between 1 and 10. The second factor is a power of 10, such as 10^3 or 10^6.

Example

The average distance between the planet Mercury and the sun is 58,000,000 km. To write the first factor in scientific notation, insert a decimal point in the original number so that you have a number between 1 and 10. In the case of 58,000,000, the number is 5.8.

To determine the power of 10, count the number of places that the decimal point moved. In this case, it moved 7 places.

$$58{,}000{,}000 \text{ km} = 5.8 \times 10^7 \text{ km}$$

Practice

Express 6,590,000 in scientific notation.